UNIT VECTORS

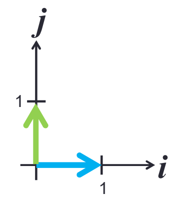
1

What is a UNIT VECTOR?

- A unit vector is a vector that is 1 unit long!

There are two standard unit vectors:

- $\vec{\imath}$ is a unit vector … $\vec{\imath} = \langle 1,0 \rangle$.
- $\vec{\jmath}$ is a unit vector … $\vec{\jmath} = \langle 0,1 \rangle$.



2

a Vector in two forms …

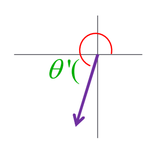
A vector in component form … $\langle 8, 26 \rangle$

… can also be written as …

… the sum of unit vectors … $8\vec{\imath} + 26\vec{\jmath}$

3

Example 1 … initial point: (-1, 5) terminal point: (-2, -3)



a) Find component form.

$\langle -2-(-1), -3-5 \rangle = \boxed{\langle -1, -8 \rangle}$

b) Write as a sum of unit vectors.

$\boxed{-\vec{\imath} - 8\vec{\jmath}}$

c) Find the magnitude.

$\sqrt{1+64} = \sqrt{65} = \boxed{8.06}$

d) Find the direction. Use [0°, 360°).

$\theta' = \tan^{-1}\left(\frac{-8}{-1}\right) = \tan^{-1} 8 = 82.87°$

$\theta = 180° + 82.87° = \boxed{262.87°}$

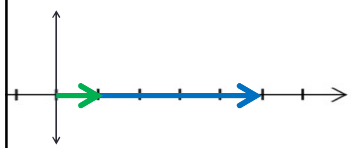
4

Example 2 … Vector Operations

- Given $\vec{v} = 3\vec{\imath} - \vec{\jmath}$ and $\vec{w} = -2\vec{\imath} + 3\vec{\jmath}$.
- Find: $\vec{v} = \langle 3,-1\rangle$ $\vec{w} = \langle -2,3\rangle$
  - a) $4\vec{v} + 2\vec{w} = 4\langle 3,-1\rangle + 2\langle -2,3\rangle$
    $= \langle 12,-4\rangle + \langle -4,6\rangle = \boxed{\langle 8,2\rangle}$ or $\boxed{8\vec{i} + 2\vec{j}}$
  - b) $\vec{v} - 3\vec{w} = \langle 3,-1\rangle - 3\langle -2,3\rangle$
    $= \langle 3,-1\rangle + \langle 6,-9\rangle = \boxed{\langle 9,-10\rangle}$ or $\boxed{9\vec{i} - 10\vec{j}}$
  - c) $\frac{1}{2}\vec{v} + \frac{1}{2}\vec{w} = \frac{1}{2}\langle 3,-1\rangle + \frac{1}{2}\langle -2,3\rangle$
    $= \left\langle \frac{3}{2}, -\frac{1}{2}\right\rangle + \left\langle -\frac{2}{2}, \frac{3}{2}\right\rangle = \left\langle \frac{1}{2}, \frac{2}{2}\right\rangle = \boxed{\left\langle \frac{1}{2}, 1\right\rangle} = \boxed{\frac{1}{2}\vec{i} + \vec{j}}$

5

Demo to help explain the new formula you are about to see.



- consider the vector $\langle 5, 0\rangle$
- it has magnitude 5
- a ***unit vector*** in the same direction would have magnitude 1
- that would be vector $\langle 1, 0\rangle$

$$\frac{\langle 5, 0\rangle}{5} = \frac{1}{5}\langle 5,0\rangle = \langle 1,0\rangle$$

6

a unit vector in the direction of $\vec{v}$ …

A unit vector, $\vec{u}$, in the direction of $\vec{v}$ …

… is given by: $\vec{u} = \dfrac{\vec{v}}{\|\vec{v}\|}$ ← original vector ← original magnitude

7

Example 3 … Find a unit vector in the direction of each given vector.

a) $\vec{v} = \langle 3, -4\rangle$

$\|\vec{v}\| = \sqrt{9+16} = \sqrt{25} = 5$

$\vec{u} = \frac{\langle 3,-4\rangle}{5}$

$\vec{u} = \frac{1}{5}\langle 3,-4\rangle$

$\vec{u} = \boxed{\left\langle \frac{3}{5}, -\frac{4}{5}\right\rangle}$

b) $-6\vec{\imath} + 4\vec{\jmath}$

$\|\vec{v}\| = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

$\vec{u} = \frac{\langle -6,4\rangle}{2\sqrt{13}} = \frac{1}{2\sqrt{13}}\langle -6,4\rangle$

$= \left\langle -\frac{6}{2\sqrt{13}}, \frac{4}{2\sqrt{13}}\right\rangle = \left\langle -\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}\right\rangle$

$\vec{u} = \boxed{\left\langle -\frac{3\sqrt{13}}{13}, \frac{2\sqrt{13}}{13}\right\rangle}$

or $\boxed{-\frac{3\sqrt{13}}{13}\vec{i} + \frac{2\sqrt{13}}{13}\vec{j}}$

8