Sum + Difference Identities - Mixed WS

10 (a) $\sin 15° = \sin(45° - 30°)$

$\sin 45 \cos 30 - \cos 45 \sin 30$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

(b) $\cos 15° = \cos(45-30)$

$\cos 45 \cos 30 + \sin 45 \sin 30$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

(c) $\tan 15° = \tan(45-30)$

$\frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1\left(\frac{\sqrt{3}}{3}\right)} = \frac{\frac{3}{3} - \frac{\sqrt{3}}{3}}{\frac{3}{3} + \frac{\sqrt{3}}{3}} = \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}}$

$= \frac{3-\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3+\sqrt{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}} \cdot \frac{3-\sqrt{3}}{3-\sqrt{3}} = \frac{9-6\sqrt{3}+3}{9-3} = \frac{12-6\sqrt{3}}{6} = \boxed{2-\sqrt{3}}$

16 (a) $\sin \frac{17\pi}{12} = \sin\left(\frac{15\pi}{12} + \frac{2\pi}{12}\right) = \sin\left(\frac{5\pi}{4} + \frac{\pi}{6}\right)$

$= \sin \frac{5\pi}{4} \cos \frac{\pi}{6} + \cos \frac{5\pi}{4} \sin \frac{\pi}{6}$

$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + -\frac{\sqrt{2}}{2} \cdot \frac{1}{2} = -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$

(b) $\cos \frac{17\pi}{12} = \cos\left(\frac{5\pi}{4} + \frac{\pi}{6}\right) = \cos \frac{5\pi}{4} \cos \frac{\pi}{6} - \sin \frac{5\pi}{4} \sin \frac{\pi}{6}$

$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - -\frac{\sqrt{2}}{2} \cdot \frac{1}{2} = -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{-\sqrt{6}+\sqrt{2}}{4}}$

(c) $\tan \frac{17\pi}{12} = \tan\left(\frac{5\pi}{4} + \frac{\pi}{6}\right) = \frac{\tan \frac{5\pi}{4} + \tan \frac{\pi}{6}}{1 - \tan \frac{5\pi}{4} \tan \frac{\pi}{6}}$

$= \frac{1 + \frac{\sqrt{3}}{3}}{1 - (1)\left(\frac{\sqrt{3}}{3}\right)} = \frac{\frac{3}{3} + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}} = \frac{\frac{3+\sqrt{3}}{3}}{\frac{3}{3} - \frac{\sqrt{3}}{3}} = \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}}$

$= \frac{3+\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3-\sqrt{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}} \cdot \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{9+6\sqrt{3}+3}{9-3}$

$= \frac{12+6\sqrt{3}}{6} = \boxed{2+\sqrt{3}}$

19. $\cos 40^\circ \cos 15^\circ - \sin 40^\circ \sin 15^\circ$

$= \cos(40+15) = \boxed{\cos(55^\circ)}$

20. $\sin 110^\circ \cos 80^\circ + \cos 110^\circ \sin 80^\circ$

$\sin(110+80) = \boxed{\sin(190^\circ)}$

21. $\sin 340^\circ \cos 50^\circ - \cos 340^\circ \sin 50^\circ$

$\sin(340-50) = \boxed{\sin(290^\circ)}$

use even/odd identity

22. $\cos 20^\circ \cos 30^\circ + \sin 20^\circ \sin 30^\circ$

$\cos(20-30) = \cos(-10) = \boxed{\cos 10^\circ}$

23. $\dfrac{\tan 325^\circ - \tan 86^\circ}{1+\tan 325^\circ \tan 86^\circ} = \tan(325-86) = \boxed{\tan(239^\circ)}$

24. $\dfrac{\tan 140^\circ - \tan 60^\circ}{1+\tan 140^\circ \tan 60^\circ} = \tan(140-60) = \boxed{\tan(80^\circ)}$

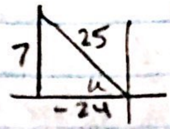
27. $\cos\frac{\pi}{7}\cos\frac{\pi}{5} - \sin\frac{\pi}{7}\sin\frac{\pi}{5} =$

$= \cos\left(\frac{\pi}{7}+\frac{\pi}{5}\right) = \cos\left(\frac{5\pi}{35}+\frac{7\pi}{35}\right) = \boxed{\cos\left(\frac{12\pi}{35}\right)}$

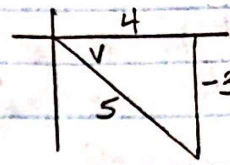
28. $\sin\frac{2\pi}{9}\cos\frac{\pi}{10} + \cos\frac{2\pi}{9}\sin\frac{\pi}{10}$

$= \sin\left(\frac{2\pi}{9}+\frac{\pi}{10}\right) = \sin\left(\frac{20\pi}{90}+\frac{9\pi}{90}\right) = \boxed{\sin\left(\frac{29\pi}{90}\right)}$

$\sin u = \frac{7}{25}, \quad \frac{\pi}{2} < u < \pi$



$\cos v = \frac{4}{5}, \quad \frac{3\pi}{2} < v < 2\pi$



39. $\cos(u+v) = \cos u \cos v - \sin u \sin v$

$= \frac{-24}{25} \cdot \frac{4}{5} - \frac{7}{25} \cdot \frac{-3}{5} = \frac{-96}{125} + \frac{21}{125} = \frac{-75}{125} = \boxed{\frac{-3}{5}}$

40. $\sin(u+v) = \sin u \cos v + \cos u \sin v$

$= \frac{7}{25} \cdot \frac{4}{5} + \frac{-24}{25} \cdot \frac{-3}{5} = \frac{28}{125} + \frac{72}{125} = \frac{100}{125} = \boxed{\frac{4}{5}}$

41. $\sin(v-u) = \sin v \cos u - \cos v \sin u$

$= -\frac{3}{5} \cdot \frac{-24}{25} - \frac{4}{5} \cdot \frac{7}{25} = \frac{72}{125} - \frac{28}{125} = \boxed{\frac{44}{125}}$

42. $\cos(u-v) = \cos u \cos v + \sin u \sin v$

$\frac{-24}{25} \cdot \frac{4}{5} + \frac{7}{25} \cdot \frac{-3}{5} = \frac{-96}{125} + \frac{-21}{125} = \boxed{\frac{-117}{125}}$

43. $\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{\frac{7}{-24} + -\frac{3}{4}}{1 - (-\frac{7}{24})(-\frac{3}{4})}$

$= \frac{\frac{-7}{24} + \frac{-18}{24}}{\frac{96}{96} - \frac{21}{96}} = \frac{\frac{-25}{24}}{\frac{75}{96}} = -\frac{\cancel{25}}{\cancel{24}} \cdot \frac{\cancel{96}^{4}}{\cancel{75}_{3}} = \boxed{-\frac{4}{3}}$

44. $\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v} = \frac{-\frac{7}{24} - -\frac{3}{4}}{1 + (-\frac{7}{24})(-\frac{3}{4})}$

$= \frac{-\frac{7}{24} + \frac{18}{24}}{\frac{96}{96} + \frac{21}{96}} = \frac{\frac{11}{24}}{\frac{117}{96}} = \frac{11}{\cancel{24}} \cdot \frac{\cancel{96}^{4}}{117} = \boxed{\frac{44}{117}}$