PreCalculus

Name Key

Solving Trig Equations with Double and Half Angles WS

Solve over the interval $[0, 2\pi)$.

1. $\cos 2x = \cos x$

$2\cos^2 x - 1 = \cos x$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0 \quad \cos x - 1 = 0$

$2\cos x = -1 \quad \cos x = 1$

$\cos x = -\frac{1}{2}$

$x = 0\pi$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

2. $\cos 2x + \cos x + 1 = 0$

$2\cos^2 x - 1 + \cos x + 1 = 0$

$2\cos^2 x + \cos x = 0$

$\cos x (2\cos x + 1) = 0$

$\cos x = 0 \quad 2\cos x + 1 = 0$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

3. $1 - \cos 2x - \sin x = 0$

$1 - (1 - 2\sin^2 x) - \sin x = 0$

$1 - 1 + 2\sin^2 x - \sin x = 0$

$2\sin^2 x - \sin x = 0$

$\sin x (2\sin x - 1) = 0$

$\sin x = 0 \quad 2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = 0\pi, \pi$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

4. $\sin^2 x + \cos 2x - \cos x = 0$

$\sin^2 x + \cos^2 x - \sin^2 x - \cos x = 0$

$1 - \sin^2 x - \cos x = 0$

$1 - (1 - \cos^2 x) - \cos x = 0$

$1 - 1 + \cos^2 x - \cos x = 0$

$\cos^2 x - \cos x = 0$

$\cos x (\cos x - 1) = 0$

$\cos x = 0 \quad \cos x - 1 = 0$

$\cos x = 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = 0\pi$

5. $\sin 2x = \cos x$

$2\sin x \cos x = \cos x$

$-\cos x \quad -\cos x$

subtract $\cos x$

$2\sin x \cos x - \cos x = 0$

$\cos x (2\sin x - 1) = 0$

$\cos x = 0 \quad 2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

6. $3\cos 2x - 5\cos x = 1$

$3(2\cos^2 x - 1) - 5\cos x = 1$

$6\cos^2 x - 3 - 5\cos x - 1 = 0$

$6\cos^2 x - 5\cos x - 4 = 0$

$(3\cos x - 4)(2\cos x + 1) = 0$

$3\cos x - 4 = 0 \quad 2\cos x + 1 = 0$

$3\cos x = 4 \quad 2\cos x = -1$

$\cos x = \frac{4}{3} \quad \cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

7. $\sin 2x \sin x + \cos 2x \cos x = 1$

$2\sin x\cos x \sin x + (1-2\sin^2 x)\cos x = 1$

$2\sin^2 x\cos x + \cos x - 2\sin^2 x\cos x = 1$

$\cos x = 1$

$\boxed{x = 0\pi}$

8. $\cos 2x + 3\cos x = 1$

$2\cos^2 x - 1 + 3\cos x = 1$

$-1 \qquad -1$

$2\cos^2 x + 3\cos x - 2 = 0$

$(2\cos x - 1)(\cos x + 2) = 0$

$2\cos x - 1 = 0 \qquad \cos x + 2 = 0$

$2\cos x = 1 \qquad \cos x = -2$

$\cos x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

9. $\sin 2x - \sin x = 0$

$2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0 \qquad 2\cos x - 1 = 0$

$\boxed{x = 0\pi, \pi} \qquad 2\cos x = 1$

$\cos x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

10. $\cos 2x + \cos x = 0$

$2\cos^2 x - 1 + \cos x = 0$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0 \qquad \cos x + 1 = 0$

$2\cos x = 1 \qquad \cos x = -1$

$\cos x = \frac{1}{2} \qquad \boxed{x = \pi}$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

11. $\cos\frac{x}{2} - \sin x = 0$

$\left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2 = (\sin x)^2$

$(2)\ \frac{1+\cos x}{2} = \sin^2 x\ (2)$

$1 + \cos x = 2\sin^2 x$

$1 + \cos x = 2(1 - \cos^2 x)$

$1 + \cos x = 2 - 2\cos^2 x$

$-2 + 2\cos^2 x \quad -2 + 2\cos^2 x$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0 \qquad \cos x + 1 = 0$

$\cos x = \frac{1}{2} \quad \cos x = -1$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3} \quad x = \pi}$

12. $\sin\frac{x}{2} + \cos x - 1 = 0$

$\left(\pm\sqrt{\frac{1-\cos x}{2}}\right)^2 = (1 - \cos x)^2$

$\frac{1-\cos x}{2} = (1-\cos x)(1-\cos x)$

$2\left(\frac{1-\cos x}{2}\right) = 2(1 - 2\cos x + \cos^2 x)$

$1 - \cos x = 2 - 4\cos x + 2\cos^2 x$

$0 = 2\cos^2 x - 3\cos x + 1$

$0 = (2\cos x - 1)(\cos x - 1)$

$2\cos = 1 \quad \cos x = \frac{1}{2} \quad \cos x = 1$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3} \quad x = 0\pi}$

Answers: 1. $0, \frac{2\pi}{3}, \frac{4\pi}{3}$ 2. $\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}$ 3. $0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$ 4. $0, \frac{\pi}{2}, \frac{3\pi}{2}$

5. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$ 6. $\frac{2\pi}{3}, \frac{4\pi}{3}$ 7. 0 8. $\frac{\pi}{3}, \frac{5\pi}{3}$ 9. $0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

10. $\pi, \frac{\pi}{3}, \frac{5\pi}{3}$ 11. $\pi, \frac{\pi}{3}, \frac{5\pi}{3}$ 12. $0, \frac{\pi}{3}, \frac{5\pi}{3}$