

Solve over  $[0, 2\pi)$ .



$\sin^2 x + \cos^2 x = 1$   
 $\tan^2 x + 1 = \sec^2 x$   
 $\cot^2 x + 1 = \csc^2 x$

1.  $\tan x = 2 \sin x$

$(\cos x) \frac{\sin x}{\cos x} = 2 \sin x (\cos x)$   
 $\sin x = 2 \sin x \cos x$   
 $0 = 2 \sin x \cos x - \sin x$   
 $0 = \sin x (2 \cos x - 1)$

$\sin x = 0$   
 $X = 0\pi, \pi$   
 $2 \cos \cos x - 1 = 0$   
 $2 \cos x = 1$   
 $\cos x = \frac{1}{2}$   
 $X = \frac{\pi}{3}, \frac{5\pi}{3}$

2.  $1 + \sin x = 2 \cos^2 x$  Pyth. Subst.

$1 + \sin x = 2(1 - \sin^2 x)$   
 $1 + \sin x = 2 - 2 \sin^2 x$   
 $2 \sin^2 x + \sin x - 1 = 0$   
 $(2 \sin x - 1)(\sin x + 1) = 0$   
 $2 \sin x - 1 = 0$      $\sin x + 1 = 0$   
 $\sin x = \frac{1}{2}$      $\sin x = -1$

$\sin x = \frac{1}{2}$   
 $X = \frac{\pi}{6}, \frac{5\pi}{6}$   
 $\sin x = -1$   
 $X = \frac{3\pi}{2}$

3.  $\sin^2 x = 2 \cos x + 2$

$1 - \cos^2 x = 2 \cos x + 2$   
 $0 = \cos^2 x + 2 \cos x + 1$   
 $(\cos x + 1)(\cos x + 1) = 0$   
 $\cos x + 1 = 0$   
 $\cos x = -1$   
 $X = \pi$

4.  $\tan x = \cot x$

$\tan x = \frac{1}{\tan x}$

$\tan^2 x = 1$   
 $\tan x = \pm 1$   
 $X = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

5.  $\csc^2 x = \cot x + 1$

$\cot^2 x + 1 = \cot x + 1$   
 $\cot^2 x - \cot x = 0$   
 $\cot x (\cot x - 1) = 0$   
 $\cot x = 0$      $\cot x - 1 = 0$   
 $\cot x = 1$

$\cot x = 0$   
 $X = \frac{\pi}{2}, \frac{3\pi}{2}$   
 $\cot x = 1$   
 $X = \frac{\pi}{4}, \frac{5\pi}{4}$

6.  $\tan^2 x = -\frac{3}{2} \sec x$

$(\sec^2 x - 1) = (-\frac{3}{2} \sec x)^2$   
 $2 \sec^2 x - 2 = -3 \sec x$   
 $2 \sec^2 x + 3 \sec x - 2 = 0$   
 $(2 \sec x - 1)(\sec x + 2) = 0$

$2 \sec x - 1 = 0$   
 $\sec x = \frac{1}{2}$   
 $\cos x = 2$   
 $\sec x + 2 = 0$   
 $\sec x = -2$   
 $\cos x = -\frac{1}{2}$   
 $X = \frac{2\pi}{3}, \frac{4\pi}{3}$

7.  $\sin x \tan x = -\tan x$

$\sin x \tan x + \tan x = 0$   
 $\tan x (\sin x + 1) = 0$   
 $\tan x = 0$      $\sin x + 1 = 0$   
 $\sin x = -1$   
 $X = 0\pi, \pi$      $X = \frac{3\pi}{2}$

8.  $2 \sin^2 x = 3 \sin x - 1$

$2 \sin^2 x - 3 \sin x + 1 = 0$   
 $(2 \sin x - 1)(\sin x - 1) = 0$   
 $2 \sin x - 1 = 0$      $\sin x - 1 = 0$   
 $\sin x = \frac{1}{2}$      $\sin x = 1$   
 $X = \frac{\pi}{6}, \frac{5\pi}{6}$      $X = \frac{\pi}{2}$

9.  $2 \sin^2 x = \sqrt{3} \sin x$

$2 \sin^2 x - \sqrt{3} \sin x = 0$   
 $\sin x (2 \sin x - \sqrt{3}) = 0$   
 $\sin x = 0$      $2 \sin x - \sqrt{3} = 0$   
 $\sin x = \frac{\sqrt{3}}{2}$   
 $X = 0\pi, \pi$      $X = \frac{\pi}{3}, \frac{2\pi}{3}$

10.  $\cot^2 x + \csc^2 x = 3$

$\cot^2 x + \cot^2 x + 1 = 3$   
 $2 \cot^2 x = 2$   
 $\cot^2 x = 1$   
 $\cot x = \pm 1$   
 $X = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$$11. 2 \cos x \csc x = \sqrt{3} \csc x$$

$$2 \cos x \csc x - \sqrt{3} \csc x = 0$$

$$\csc x (2 \cos x - \sqrt{3}) = 0$$

$$\csc x = 0$$

$$2 \cos x - \sqrt{3} = 0$$

$$\sin x = \frac{1}{0}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{11\pi}{6}$$

$$12. 3 \cos x + 3 = 2 \sin^2 x$$

$$3 \cos x + 3 = 2(1 - \cos^2 x)$$

$$3 \cos x + 3 = 2 - 2 \cos^2 x$$

$$2 \cos^2 x + 3 \cos x + 1 = 0$$

$$(2 \cos x + 1)(\cos x + 1) = 0$$

$$\cos x = -\frac{1}{2} \quad \cos x = -1$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$x = \pi$$

$$13. \tan^2 x = \sqrt{3} \tan x$$

$$\tan^2 x - \sqrt{3} \tan x = 0$$

$$\tan x (\tan x - \sqrt{3}) = 0$$

$$\tan x = 0$$

$$\tan x - \sqrt{3} = 0$$

$$x = 0\pi, \pi$$

$$\tan x = \sqrt{3}$$

$$x = \frac{\pi}{3}, \frac{4\pi}{3}$$

$$14. (\tan x - 1)(\sec x - 1) = 0$$

$$\tan x - 1 = 0 \quad \sec x - 1 = 0$$

$$\tan x = 1$$

$$\sec x = 1$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$\cos x = 1$$

$$x = 0\pi$$

$$15. \sec^2 x - 2 \tan x = 0$$

$$1 + \tan^2 x - 2 \tan x = 0$$

$$\tan^2 x - 2 \tan x + 1 = 0$$

$$(\tan x - 1)(\tan x - 1) = 0$$

$$\tan x - 1 = 0$$

$$\tan x = 1$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$16. (\sin^2 x - 1)(\tan x + 1) = 0$$

$$\sin^2 x - 1 = 0 \quad \tan x + 1 = 0$$

$$\sin^2 x = 1$$

$$\tan x = -1$$

$$\sin x = \pm 1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$17. 3 \cos x + \sqrt{2} = \cos x$$

$$2 \cos x = -\sqrt{2}$$

$$\cos x = -\frac{\sqrt{2}}{2}$$

$$x = \frac{3\pi}{4}, \frac{5\pi}{4}$$

$$18. (\sec^2 x - 2)(\csc x + 1) = 0$$

$$\sec^2 x = 2$$

$$\csc x = -1$$

$$\cos^2 x = \frac{1}{2}$$

$$\sin x = -1$$

$$\cos x = \pm \frac{\sqrt{2}}{2}$$

$$x = \frac{3\pi}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$19. \cot x (\csc x + 2) = 0$$

$$\cot x = 0$$

$$\csc x + 2 = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\csc x = -2$$

$$\sin x = -\frac{1}{2}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$20. 2 \cos^2 x - 7 \cos x = -3$$

$$2 \cos^2 x - 7 \cos x + 3 = 0$$

$$(2 \cos x - 1)(\cos x - 3) = 0$$

$$2 \cos x - 1 = 0$$

$$\cos x - 3 = 0$$

$$\cos x = \frac{1}{2}$$

$$\cos x \neq 3$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$



↙ Double

$$21. 6\sin 2x - 3 = 0$$

$$6 \frac{\sin 2x}{6} = \frac{3}{6}$$

$$\sin 2x = \frac{1}{2}$$

$$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

$$\frac{\pi}{6} + \frac{12\pi}{6} = \frac{13\pi}{6}$$

$$\frac{5\pi}{6} + \frac{12\pi}{6} = \frac{17\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

↙ Triple

$$22. \tan 3x(\tan x - 1) = 0$$

$$\tan 3x = 0 \quad \tan x - 1 = 0$$

$$\tan x = 1$$

$$3x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$$

$$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$23. 3\tan^2 2x = 1$$

$$\sqrt{\tan^2 2x} = \sqrt{\frac{1}{3}}$$

$$\tan 2x = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$$

$$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$$

$$24. 4\sec 3x + 8 = 0$$

$$4\sec 3x = -8$$

$$\sec 3x = -2$$

$$\cos 3x = -\frac{1}{2}$$

$$3x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}$$

$$x = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \frac{14\pi}{9}, \frac{16\pi}{9}$$