Sine and Cosine Graphs with Translations

Sine and Cosine Graph with Translations

In general: $y = \pm a \cdot \sin(bx - c) + d$

- neg: reflects the x-axis; amplitude: $|a|$; vertical shrink/stretch
- period: $\frac{2\pi}{b}$ or $\frac{360^\circ}{b}$
- horizontal/phase shift $\leftrightarrow$: $bx - c = 0$, $b\theta - c = 0^\circ$
- vertical shift $\updownarrow$: + / −

$y = \pm a \cdot \cos(bx - c) + d$

Domain: [phase shift, $bx - c = 2\pi$, $b\theta - c = 360^\circ$]

Graphing Prep and Checklist:

- Domain: one phase shift
  - > $b\theta - c = 0^\circ$ and $b\theta - c = 360^\circ$ or $[b\theta - c = 0^\circ, b\theta - c = 360^\circ]$
  - > $bx - c = 0$ and $bx - c = 2\pi$ $[bx - c = 0, bx - c = 2\pi]$
- Vertical Shift: $\pm$ d $\updownarrow^{+}_{-}$
- Amplitude: |a|
- Cosine or Sine with a Reflection? if "a" is negative, reflect
- Period:
  - > $\frac{360^\circ}{b}$ or $\frac{2\pi}{b}$
  - > Look at domain: right - left
- Phase Shift:
  - > $b\theta - c = 0^\circ$ or $bx - c = 0$
  - > Left side of domain
- Range: Think about amplitude with vertical shift

$y = \sin x$ $y = -\sin x$ $y = \cos x$ $y = -\cos x$

$y = a\sin(bx - c) + d$

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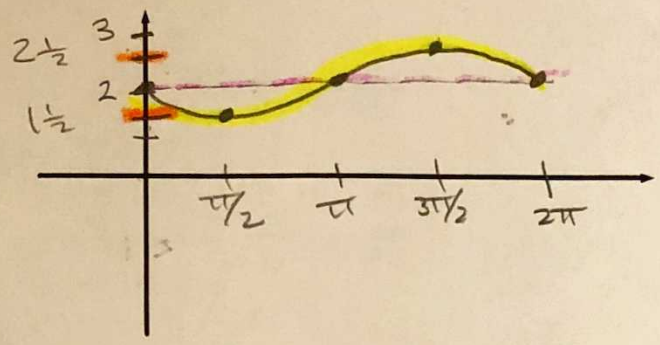
* No "c" value

*Reflect

1) $y = -\frac{1}{2}\sin x + 2$



domain: $[0, 2\pi]$

vertical shift: 2 (up 2)

amplitude: $|-\frac{1}{2}| = \frac{1}{2}$

period: 2π → $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$ → Domain $2\pi - 0 = 2\pi$

phase shift: None

range: $[1\frac{1}{2}, 2\frac{1}{2}]$

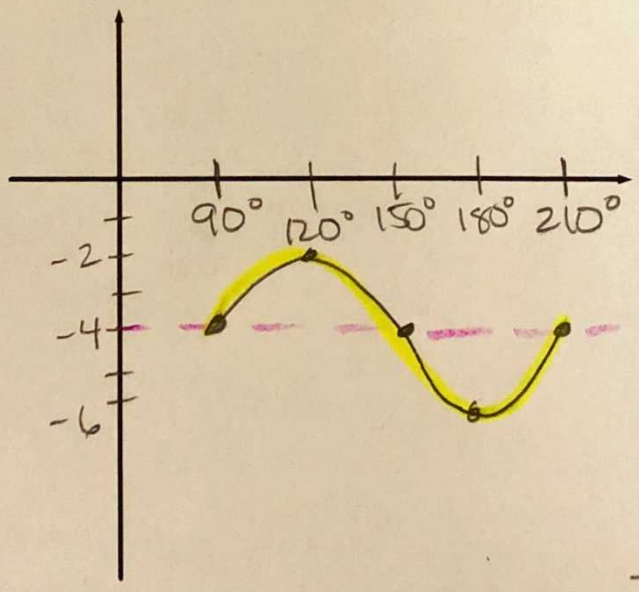
D: $[bx - c = 0,\ bx - c = 2\pi]$

$[x = 0,\ x = 2\pi]$

$[0, 2\pi]$

$y = a\sin(b\theta - c) + d$

2) $y = 2\sin(3\theta - 270°) - 4$



domain: $[90°, 210°]$

vertical shift: -4 (down 4)

amplitude: 2

period: 120° → $\frac{360°}{3} = 120°$ → $210° - 90° = 120°$

phase shift: 90°

range: $[-6, -2]$

$\frac{90° + 210°}{2} = 150°$

$\frac{90° + 150°}{2} = 120°$

$\frac{150° + 210}{2} = 180°$

D: $[b\theta - c = 0°,\ b\theta - c = 360°]$

$[3\theta - 270° = 0°,\ 3\theta - 270° = 360°]$

$3\theta = 270°$ $\quad 3\theta = 630°$

$\theta = 90°$ $\quad \theta = 210°$

$[90°, 210°]$

$y = a\sin(bx-c)+d$

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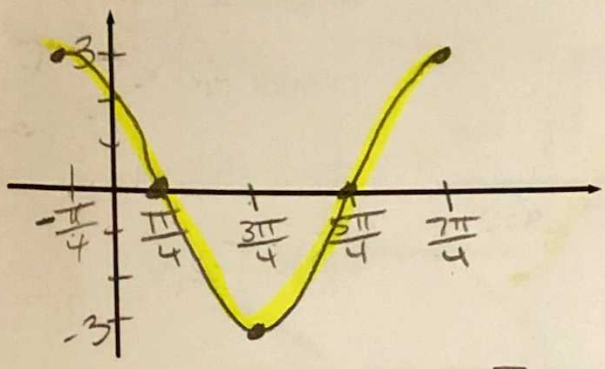
3) $y = 3\cos\left(x+\frac{\pi}{4}\right)$



domain: $\left[-\frac{\pi}{4}, \frac{7\pi}{4}\right]$

vertical shift: None ("d" value)

amplitude: 3

period: 2π $\quad \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$ $\quad \frac{7\pi}{4} - -\frac{\pi}{4} = \frac{8\pi}{4}$

phase shift: $-\frac{\pi}{4}$

range: $[-3, 3]$

D: $[bx-c=0,\ bx-c=2\pi]$

$\left[x+\frac{\pi}{4}=0,\ x+\frac{\pi}{4}=2\pi\right]$

$x=-\frac{\pi}{4}$ $\quad x=2\pi-\frac{\pi}{4}$

$x=\frac{8\pi}{4}-\frac{\pi}{4}$

D: $\left[-\frac{\pi}{4}, \frac{7\pi}{4}\right]$

$-\frac{\pi}{4}$ $\frac{\pi}{4}$ $\frac{3\pi}{4}$ $\frac{5\pi}{4}$ $\frac{7\pi}{4}$

$\frac{-\frac{\pi}{4}+\frac{7\pi}{4}}{2} = \frac{6\pi}{4}\cdot\frac{1}{2} = \frac{6\pi}{8} = \frac{3\pi}{4}$

$\frac{-\frac{\pi}{4}+\frac{3\pi}{4}}{2} = \frac{2\pi}{4}\cdot\frac{1}{2} = \frac{2\pi}{8} = \frac{\pi}{4}$

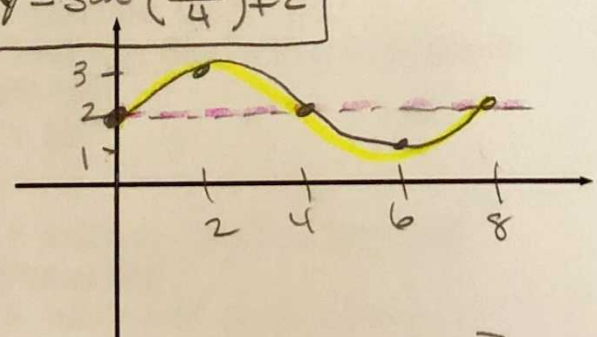
$\frac{\frac{3\pi}{4}+\frac{7\pi}{4}}{2} = \frac{10\pi}{4}\cdot\frac{1}{2} = \frac{10\pi}{8} = \frac{5\pi}{4}$

$y = a\sin(bx-c)+d$

4) $y = 2-\sin\left(-\frac{\pi x}{4}\right)$

Rewrite → $y = -\sin\left(-\frac{\pi x}{4}\right)+2$

change argument → $y = \sin\left(\frac{\pi x}{4}\right)+2$



domain: $[0, 8]$

vertical shift: 2

amplitude: 1

period: 8 $\quad \frac{2\pi}{\pi/4} = 2\pi\cdot\frac{4}{\pi} = 8$ $\quad 8-0=8$

phase shift: None ("c" value)

range: $[1, 3]$

D: $[bx-c=0,\ bx-c=2\pi]$

$\left[\frac{4}{\pi}\frac{\pi x}{4} = 0,\ \frac{4}{\pi}\frac{\pi x}{4} = 2\pi\right]\left(\frac{4}{\pi}\right)$

$[x=0,\ x=8]$

$[0, 8]$

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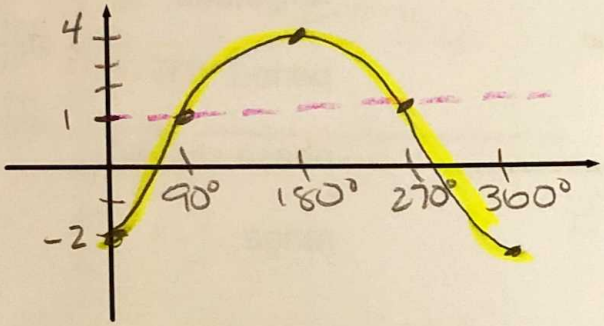
$y = a\cos(b\theta - c) + d$

5) $y = 1 - 3\cos\theta$

$y = -3\cos\theta + 1$



D: $[b\theta - c = 0°,\ b\theta - c = 360°]$

$[\theta = 0°,\ \theta = 360°]$

$[0°, 360°]$

domain: $[0°, 360°]$

vertical shift: 1

amplitude: $|-3| = 3$

period: $360°$ $\frac{360°}{b} = \frac{360°}{1}$ $360° - 0° = 360°$

phase shift: None (No "C" value)

range: $[-2, 4]$