PreCalculus

Name Key

**Simplifying and Verifying with Sum & Difference Identities WS**

**#1-7. Simplify each of the following. Remember—first step is always to EXPAND**

1. $\cos\left(\theta-\frac{3\pi}{2}\right)$

$= \cos\theta\cos\frac{3\pi}{2}+\sin\theta\sin\frac{3\pi}{2}$

$= (\cos\theta)(0)+(\sin\theta)(-1) = \boxed{-\sin\theta}$

2. $\tan(\theta+\pi) = \frac{\tan\theta+\tan\pi}{1-\tan\theta\tan\pi} = \frac{\tan\theta+0}{1-\tan\theta(0)} = \frac{\tan\theta}{1} = \boxed{\tan\theta}$

$\tan\pi = 0$

3. $\cos(\pi-\theta)+\sin\left(\frac{\pi}{2}+\theta\right)$

$= \cos\pi\cos\theta+\sin\pi\sin\theta+\sin\frac{\pi}{2}\cos\theta+\cos\frac{\pi}{2}\sin\theta$

$= -1\cdot\cos\theta+0\cdot\sin\theta+1\cdot\cos\theta+0\cdot\sin\theta$

$= -\cos\theta+\cos\theta = \boxed{0}$

4. $\sin(\theta+\pi)+\cos\left(\theta-\frac{\pi}{2}\right)$

$= \sin\theta\cos\pi+\cos\theta\sin\pi+\cos\theta\cos\frac{\pi}{2}+\sin\theta\sin\frac{\pi}{2}$

$= \sin\theta(-1)+\cos\theta(0)+\cos\theta(0)+\sin\theta(1)$

$= -\sin\theta+\sin\theta = \boxed{0}$

5. $\tan(\theta+\pi)-\tan(\pi-\theta)$

$= \frac{\tan\theta+\tan\pi}{1-\tan\theta\tan\pi}-\frac{\tan\pi-\tan\theta}{1+\tan\pi\tan\theta}$

$= \frac{\tan\theta+0}{1-\tan\theta(0)}-\frac{0-\tan\theta}{1+(0)\tan\theta} = \frac{\tan\theta}{1}-\frac{-\tan\theta}{1} = \tan\theta+\tan\theta = \boxed{2\tan\theta}$

6. $\sin\left(\theta+\frac{\pi}{4}\right)+\sin\left(\theta-\frac{\pi}{4}\right)$

$= \sin\theta\cos\frac{\pi}{4}+\cos\theta\sin\frac{\pi}{4}+\sin\theta\cos\frac{\pi}{4}-\cos\theta\sin\frac{\pi}{4}$

$= \sin\theta\left(\frac{\sqrt{2}}{2}\right)+\cos\theta\left(\frac{\sqrt{2}}{2}\right)+\sin\theta\left(\frac{\sqrt{2}}{2}\right)-\cos\theta\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{2}\sin\theta+\frac{\sqrt{2}}{2}\sin\theta = \frac{2\sqrt{2}}{2}\sin\theta = \boxed{\sqrt{2}\sin\theta}$

7. $\cos\left(\theta+\frac{\pi}{4}\right)-\cos\left(\theta-\frac{\pi}{4}\right)$ watch the minus sign!

$= \cos\theta\cos\frac{\pi}{4}-\sin\theta\sin\frac{\pi}{4}-\left[\cos\theta\cos\frac{\pi}{4}+\sin\theta\sin\frac{\pi}{4}\right]$

$= -\sin\theta\left(\frac{\sqrt{2}}{2}\right)-\sin\theta\left(\frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{2}}{2}\sin\theta-\frac{\sqrt{2}}{2}\sin\theta = -\frac{2\sqrt{2}}{2}\sin\theta = \boxed{-\sqrt{2}\sin\theta}$

#8-13. Verify the following identities.

8. $\cos(\pi-\theta)+\sin\left(\frac{\pi}{2}+\theta\right)=0$

$= \cos\pi\cos\theta + \sin\pi\sin\theta + \sin\frac{\pi}{2}\cos\theta + \cos\frac{\pi}{2}\sin\theta$

$= (-1)\cos\theta + (0)\sin\theta + (1)\cos\theta + (0)\sin\theta$

$= -\cos\theta + \cos\theta = \boxed{0}$ ✓

9. $\sin(\theta+\pi)+\cos\left(\theta-\frac{\pi}{2}\right)=0$

$\sin\theta\cos\pi + \cos\theta\sin\pi + \cos\theta\cos\frac{\pi}{2} + \sin\theta\sin\frac{\pi}{2}$

$= \sin\theta(-1) + \cos\theta(0) + \cos\theta(0) + \sin\theta(1)$

$= -\sin\theta + \sin\theta = \boxed{0}$ ✓

10. $\sin(x+y)+\sin(x-y)=2\sin x\cos y$

$\sin x\cos y + \cos x\sin y + \sin x\cos y - \cos x\sin y$ (cancel out)

$= \boxed{2\sin x\cos y}$ ✓

11. $\cos(x+y)+\cos(x-y)=2\cos x\cos y$

$= \cos x\cos y - \sin x\sin y + \cos x\cos y + \sin x\sin y$ (cancel out)

$= \boxed{2\cos x\cos y}$ ✓

12. $\tan(x+\pi)-\tan(\pi-x)=2\tan x$

$= \left[\frac{\tan x+\tan\pi}{1-\tan x\tan\pi}\right] - \left[\frac{\tan\pi-\tan x}{1+\tan\pi\tan x}\right]$

$= \left[\frac{\tan x+0}{1-\tan x(0)}\right] - \left[\frac{0-\tan x}{1+(0)\tan x}\right] = \frac{\tan x}{1} - \frac{-\tan x}{1} = \boxed{2\tan x}$ ✓

13. $\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1-\tan\theta}{1+\tan\theta}$

$= \frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta} = \frac{1-\tan\theta}{1+(1)\tan\theta} = \boxed{\frac{1-\tan\theta}{1+\tan\theta}}$ ✓