******Please READ: All of the Half Angle Problems can be found AFTER the Double Angle Problems.

Review WS - Double Angle Identities

1) $\tan 450^\circ = \tan(2 \cdot 225^\circ) = \dfrac{2\tan 225^\circ}{1-\tan^2 225^\circ} = \dfrac{2(1)}{1-1^2} = \dfrac{2}{1-1}$

$\dfrac{2}{0} = \boxed{\text{undefined}}$

2) $\cos\dfrac{8\pi}{3} = \cos\left(2\cdot\dfrac{4\pi}{3}\right) = \cos^2\left(\dfrac{4\pi}{3}\right) - \sin^2\left(\dfrac{4\pi}{3}\right)$

$= \left(-\dfrac{1}{2}\right)^2 - \left(-\dfrac{\sqrt{3}}{2}\right)^2 = \dfrac{1}{4} - \dfrac{3}{4} = -\dfrac{2}{4} = \boxed{-\dfrac{1}{2}}$

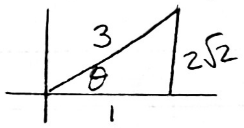
3) $\csc 600^\circ = \csc(2\cdot 300^\circ)$ *Change to sin then flip for csc

$\sin(2\cdot 300^\circ) = 2\sin 300^\circ \cos 300^\circ$

$= 2\left(-\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{2}\right) = -\dfrac{2\sqrt{3}}{4} = -\dfrac{\sqrt{3}}{2}$

$\csc 600^\circ = -\dfrac{2}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}} = \boxed{\dfrac{-2\sqrt{3}}{3}}$

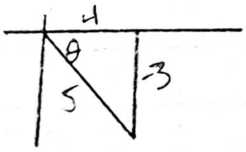
8) $\cos\theta = \dfrac{1}{3}$



$1^2 + b^2 = 3^2$
$b^2 = 8$
$b = 2\sqrt{2}$

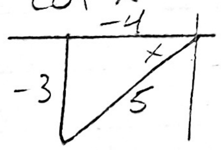
$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(\dfrac{2\sqrt{2}}{3}\right)\left(\dfrac{1}{3}\right) = \boxed{\dfrac{4\sqrt{2}}{9}}$

9) $\cos\theta = \dfrac{4}{5}$



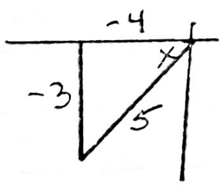
$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(-\dfrac{3}{5}\right)\left(\dfrac{4}{5}\right) = \boxed{\dfrac{-24}{25}}$

14) $\cot x = \dfrac{4}{3}$ adj / opp



$\sin 2x = 2\sin x\cos x$
$= 2\left(-\dfrac{3}{5}\right)\left(-\dfrac{4}{5}\right) = \boxed{\dfrac{24}{25}}$

15) $\cot x = \dfrac{4}{3}$ adj / opp



$\cot 2x$ *Change to tan then flip for cot

$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x} = \dfrac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2}$

$= \dfrac{\frac{6}{4}}{1-\frac{9}{16}} = \dfrac{\frac{3}{2}}{\frac{16}{16}-\frac{9}{16}} = \dfrac{\frac{3}{2}}{\frac{7}{16}} = \dfrac{3}{2}\cdot\dfrac{16}{7} = \dfrac{24}{7}$

$\boxed{\cot 2x = \dfrac{7}{24}}$

19) $\cos 2x + \sin x = -2$

$$1 - 2\sin^2 x + \sin x = -2$$
$$+2 \qquad\qquad +2$$

$$-2\sin^2 x + \sin x + 3 = 0$$ * Multiply all terms by -1

$$2\sin^2 x - \sin x - 3 = 0$$
$$(2\sin x - 3)(\sin x + 1) = 0$$
$$2\sin x - 3 = 0 \qquad \sin x + 1 = 0$$
$$2\sin x = 3 \qquad \sin x = -1$$
$$\sin x = \cancel{\tfrac{3}{2}} \qquad \boxed{x = \tfrac{3\pi}{2}}$$

20) $\cos 2x - \sin 2x = -2\sin x \cos x$

$$2\cos^2 x - 1 - \cancel{2\sin x \cos x} = -\cancel{2\sin x \cos x}$$
$$+\cancel{2\sin x \cos x} \qquad +\cancel{2\sin x \cos x}$$

$$2\cos^2 x - 1 = 0$$
$$2\cos^2 x = 1$$
$$\sqrt{\cos^2 x} = \sqrt{\tfrac{1}{2}}$$
$$\cos x = \pm\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

$$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$$

21) $\cos^2 x - \frac{3}{2}\cos 2x = 0$

$$\cos^2 x - \frac{3}{2}(2\cos^2 x - 1) = 0$$
$$\cos^2 x - 3\cos^2 x + \frac{3}{2} = 0$$
$$-2\cos^2 x + \frac{3}{2} = 0$$
$$\left(-\tfrac{1}{2}\right) - 2\cos^2 x = -\frac{3}{2}\left(-\tfrac{1}{2}\right)$$
$$\sqrt{\cos^2 x} = \sqrt{\tfrac{3}{4}}$$
$$\cos x = \pm\frac{\sqrt{3}}{2}$$

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

24) $\cos 2x - 11\cos x = 5$

$2\cos^2 x - 1 - 11\cos x = 5$
$\quad -5 \qquad\qquad -5$

$2\cos^2 x - 11\cos x - 6 = 0$

$(2\cos x + 1)(\cos x - 6) = 0$

$2\cos x + 1 = 0 \qquad \cos x - 6 = 0$

$2\cos x = -1 \qquad \cos x = \cancel{6}$

$\cos x = -\frac{1}{2}$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$

25) $\cos^2 \frac{3\pi}{7} - \sin^2 \frac{3\pi}{7} = \cos\left(2 \cdot \frac{3\pi}{7}\right) = \boxed{\cos \frac{6\pi}{7}}$

26) $\frac{2\tan 31^\circ}{1 - \tan^2 31^\circ} = \tan(2 \cdot 31^\circ) = \boxed{\tan 62^\circ}$

29) $\sin 2x = \tan x\,(1 + \cos 2x)$

$= \frac{\sin x}{\cos x}\,(\cancel{1} + 2\cos^2 x - \cancel{1})$

$= \frac{\sin x}{\cancel{\cos x}} \cdot 2\cos^{\cancel{2}} x$

$= \sin x \cdot 2\cos x$

$= 2\sin x \cos x$

$= \boxed{\sin 2x}$ ✓

30) $\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$

$= \frac{1 - \tan^2 x}{\sec^2 x}$

$= \frac{1}{\sec^2 x} - \frac{\tan^2 x}{\sec^2 x}$

$= \cos^2 x - \frac{\sin^2 x / \cos^2 x}{1 / \cos^2 x}$

$= \cos^2 x - \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1}$

$= \cos^2 x - \sin^2 x$

$= \boxed{\cos 2x}$ ✓

4) $\sin 165°$ * $165(2)=330°$ * Q2 sin is Pos

$$\sin\left(\frac{330}{2}\right)=\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}=\sqrt{\frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{2}}=\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}}$$

$$=\sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}}=\sqrt{\frac{2-\sqrt{3}}{4}}=\boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$$

5) $\cos\frac{7\pi}{8}$ * $\frac{7\pi}{8}\cdot 2=\frac{14\pi}{8}=\frac{7\pi}{4}$ * Q2 cos is neg.

$$\cos\left(\frac{7\pi/4}{2}\right)=-\sqrt{\frac{1+\cos\frac{7\pi}{4}}{2}}=-\sqrt{\frac{1+\sqrt{2}/2}{2}}=-\sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{2}}$$

$$=-\sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}}=-\sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}}=-\sqrt{\frac{2+\sqrt{2}}{4}}=\boxed{\frac{-\sqrt{2+\sqrt{2}}}{2}}$$

6) $\sec\frac{5\pi}{12}$ => $\frac{5\pi}{12}\cdot 2=\frac{10\pi}{12}=\frac{5\pi}{6}$ Q1 - All Positive

$\sec\left(\frac{5\pi/6}{2}\right)$ Find cos then take reciprocal for sec

$$\cos\left(\frac{5\pi/6}{2}\right)=\sqrt{\frac{1+\cos\frac{5\pi}{6}}{2}}=\sqrt{\frac{1+\frac{-\sqrt{3}}{2}}{2}}=\sqrt{\frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{2}}$$

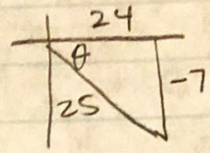
$$=\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}}=\sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}}=\sqrt{\frac{2-\sqrt{3}}{4}}$$

$$\sec\left(\frac{5\pi/6}{2}\right)=\sqrt{\frac{4}{2-\sqrt{3}}}=\sqrt{\frac{4}{2-\sqrt{3}}\cdot\frac{(2+\sqrt{3})}{(2+\sqrt{3})}}=\sqrt{\frac{8+4\sqrt{3}}{4-3}}$$

$$=\sqrt{\frac{8+4\sqrt{3}}{1}}=\frac{\sqrt{8+4\sqrt{3}}}{\sqrt{1}}=\boxed{\sqrt{8+4\sqrt{3}}}$$ or keep going ☺

$$=\sqrt{4(2+\sqrt{3})}=\boxed{2\sqrt{2+\sqrt{3}}}$$

7) $\sin\theta = -\frac{7}{25}$ $\quad \frac{270°}{2} < \frac{\theta}{2} < \frac{360°}{2}$ $\quad$ Q4



$135° < \frac{\theta}{2} < 180°$ $\quad$ Q2

↳ Cos = Neg

$\cos\frac{\theta}{2} = -\sqrt{\frac{1+\cos\theta}{2}}$

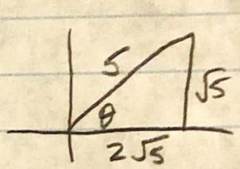
$= -\sqrt{\frac{1+24/25}{2}} = -\sqrt{\frac{25/25 + \frac{24}{25}}{2}} = -\sqrt{\frac{49/25}{2}} = -\sqrt{\frac{49}{25} \cdot \frac{1}{2}}$

$= -\sqrt{\frac{49}{50}} = -\frac{7}{\sqrt{50}} = -\frac{7}{5\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{-7\sqrt{2}}{10}}$

10) $\cos\theta = \frac{2\sqrt{5}}{5}$ $\quad 0° < \theta < 90°$ $\quad$ Q1

$0° < \frac{\theta}{2} < 45°$ $\quad$ Q1

All Pos.



$(2\sqrt{5})^2 + b^2 = 5^2$

$20 + b^2 = 25$

$b^2 = 5$

$b = \sqrt{5}$

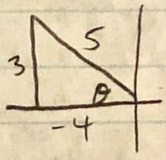
$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$

$\sqrt{\frac{1-2\sqrt{5}/5}{2}} = \sqrt{\frac{\frac{5}{5} - \frac{2\sqrt{5}}{5}}{2}} = \sqrt{\frac{\frac{5-2\sqrt{5}}{5}}{2}} = \sqrt{\frac{5-2\sqrt{5}}{5} \cdot \frac{1}{2}} = \sqrt{\frac{5-2\sqrt{5}}{10}}$

$= \frac{\sqrt{5-2\sqrt{5}}}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10(5-2\sqrt{5})}}{10} = \boxed{\frac{\sqrt{50-20\sqrt{5}}}{10}}$

11) $\cos\theta = -\frac{4}{5}$ $\quad 90° < \theta < 180°$ $\quad$ Q2

$45° < \frac{\theta}{2} < 90°$ $\quad$ Q1



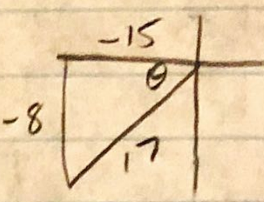
$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$

$\sqrt{\frac{1 - -4/5}{2}} = \sqrt{\frac{5/5 + 4/5}{2}} = \sqrt{\frac{9/5}{2}} = \sqrt{\frac{9}{5} \cdot \frac{1}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$

$= \boxed{\frac{3\sqrt{10}}{10}}$

12) $\cos\theta = -\frac{15}{17}$ $\quad 180^\circ < \theta < 270^\circ$

$90^\circ < \frac{\theta}{2} < 135^\circ$

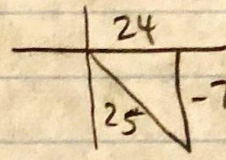


$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{1-\left(-\frac{15}{17}\right)}{-8/17} = \frac{\frac{17}{17}+\frac{15}{17}}{-8/17} = \frac{32/17}{-8/17}$$

$$= \frac{\cancel{32}^{4}}{17} \cdot -\frac{17}{\cancel{8}} = \boxed{-4}$$

13) $\tan x = -\frac{7}{24}$ $\quad \frac{3\pi}{2} < x < 2\pi$

$\frac{3\pi}{4} < \frac{x}{2} < \pi$

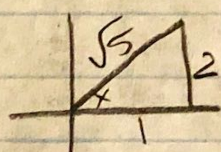


$$\cot\left(\frac{x}{2}\right) \Rightarrow \tan\left(\frac{x}{2}\right) = \frac{1-\cos x}{\sin x} = \frac{1-\frac{24}{25}}{-\frac{7}{25}} = \frac{\frac{25}{25}-\frac{24}{25}}{-\frac{7}{25}}$$

$$= \frac{\frac{1}{25}}{-7/25} = \frac{1}{25} \cdot \frac{-25}{7} = -\frac{1}{7} \text{ so } \cot\left(\frac{x}{2}\right) = \boxed{-7}$$

16) $\tan x = \frac{2}{1}$ $\quad 0 < x < \frac{\pi}{2}$

$0 < \frac{x}{2} < \frac{\pi}{4}$



$2^2 + 1^2 = c^2$

$5 = c^2$

$\sqrt{5} = c$

Q1: All Positive

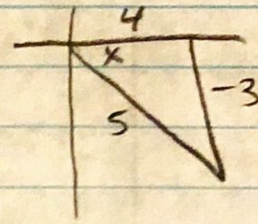
$$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}}$$

$$\sqrt{\frac{1-1/\sqrt{5}}{2}} = \sqrt{\frac{\frac{\sqrt{5}}{\sqrt{5}}-\frac{1}{\sqrt{5}}}{2}} = \sqrt{\frac{\frac{\sqrt{5}-1}{\sqrt{5}}}{2}} = \sqrt{\frac{\sqrt{5}-1}{\sqrt{5}}\cdot\frac{1}{2}} = \sqrt{\frac{\sqrt{5}-1}{2\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}}$$

$$= \sqrt{\frac{\sqrt{5}(\sqrt{5}-1)}{10}} = \sqrt{\frac{5-\sqrt{5}}{10}} = \frac{\sqrt{5-\sqrt{5}}}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10(5-\sqrt{5})}}{10} = \boxed{\frac{\sqrt{50-10\sqrt{5}}}{10}}$$

17) $\sin x = -\frac{3}{5}$ $\qquad \frac{3\pi}{2} < x < 2\pi$

$\frac{3\pi}{4} < \frac{x}{2} < \pi$

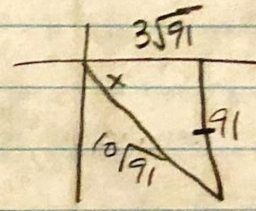


$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$

$= \frac{1-\frac{4}{5}}{-3/5} = \frac{\frac{5}{5}-\frac{4}{5}}{-3/5} = \frac{1/5}{-3/5} = \frac{1}{5}\cdot -\frac{5}{3} = \boxed{-\frac{1}{3}}$

18) $\cot x = \frac{-3\sqrt{91}}{91}$ $\frac{\text{adj}}{\text{opp}}$ $\qquad \frac{3\pi}{2} < x < 2\pi$

$\frac{3\pi}{4} < \frac{x}{2} < \pi$ Q2



$(3\sqrt{91})^2 + (-91)^2 = c^2$

$819 + 8281 = c^2$

$9100 = c^2$

$91 \quad 100$ $\quad c = 10\sqrt{91}$

$10 \quad 10$

$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}}$

$\sqrt{\frac{1-\left(3\sqrt{91}/10\sqrt{91}\right)}{2}} = \sqrt{\frac{1-\frac{3}{10}}{2}}$

$= \sqrt{\frac{\frac{10}{10}-\frac{3}{10}}{2}} = \sqrt{\frac{\frac{7}{10}}{2}} = \sqrt{\frac{7}{10}\cdot\frac{1}{2}}$

$= \sqrt{\frac{7}{20}} = \frac{\sqrt{7}}{\sqrt{20}} = \frac{\sqrt{7}}{2\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{\sqrt{35}}{10}}$

22) $2\sin\frac{x}{2} = \sin x$

$\left(2\sqrt{\frac{1-\cos x}{2}}\right)^2 = (\sin x)^2$

${}^{2}\cancel{4}\left(\frac{1-\cos x}{\cancel{2}}\right) = \sin^2 x$

$2(1-\cos x) = \sin x^2$

$2 - 2\cos x = 1 - \cos^2 x$

$-1 + \cos^2 x \qquad -1 + \cos^2 x$

$\cos^2 x - 2\cos x + 1 = 0$

$(\cos x - 1)(\cos x - 1) = 0$

$\cos x - 1 = 0 \qquad \cos x = 1 \qquad \boxed{x = 0\pi}$

23) $\sin^2 \frac{x}{2} = \cos^2 \frac{x}{2}$

$$\left(\cancel{\sqrt{\frac{1-\cos x}{2}}}\right)^{\cancel{2}} = \left(\cancel{\sqrt{\frac{1+\cos x}{2}}}\right)^{\cancel{2}}$$

$$(\cancel{2})\ \frac{1-\cos x}{\cancel{2}} = \frac{1+\cos x}{\cancel{2}}\ (\cancel{2})$$

$$1-\cos x = 1+\cos x$$
$$+\cos x \qquad +\cos x$$

$$1 = 1 + 2\cos x$$
$$-1 \quad -1$$

$$0 = 2\cos x$$
$$0 = \cos x$$

$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$$

27) $\sqrt{\frac{1-\cos\frac{\pi}{9}}{2}}$ $\quad * \sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$

$\hookrightarrow \sin\left(\frac{\pi/9}{2}\right) = \sin\frac{\pi}{9}\cdot\frac{1}{2} = \boxed{\sin\frac{\pi}{18}}$

28) $\frac{1-\cos 80^\circ}{\sin 80^\circ}$ $\quad * \tan\frac{u}{2} = \frac{1-\cos u}{\sin u}$

$\hookrightarrow \tan\frac{80^\circ}{2} = \boxed{\tan 40^\circ}$