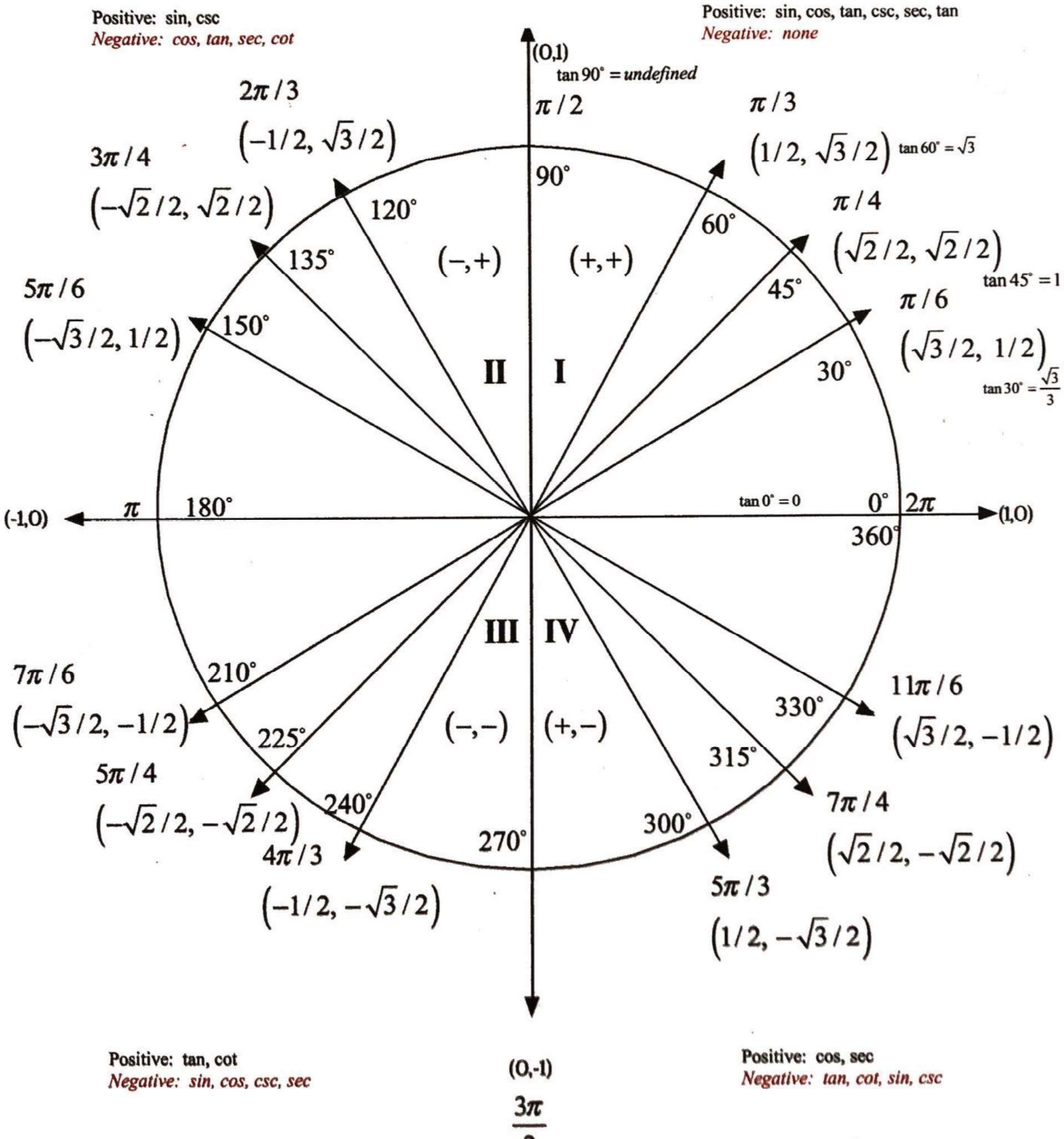
Angles and Radians of a Unit Circle



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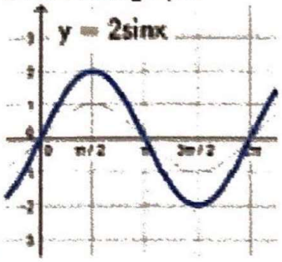
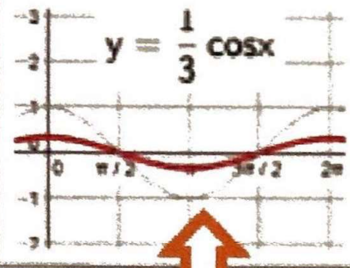
Translating Periodic Function

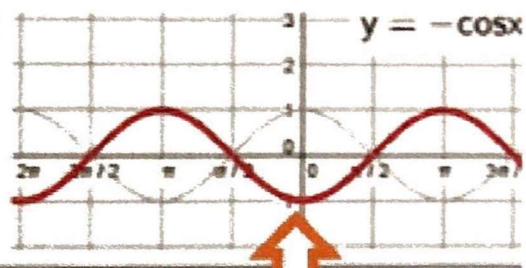
Amplitude

The amplitude, a, vertically stretches or shrinks the graph.





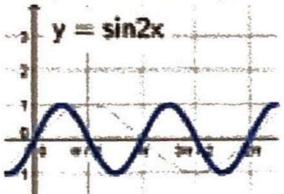
When 0 < **a** < 1, the function is *shrunk vertically*

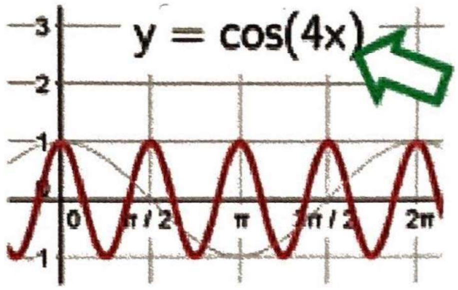


When **a** < 0, the function is *reflected across the x-axis*

Period

$\frac{2\pi}{b}$ is the period of the curve. The period is the interval required to complete one full cycle.



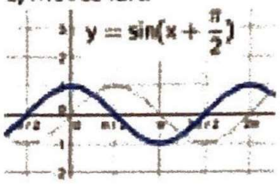


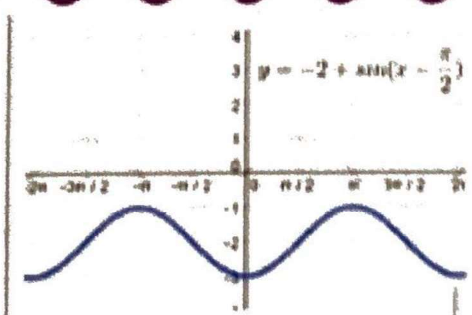
4 times as many oscillations in what is normally 1 period (2π). So $\frac{2\pi}{4} = \frac{\pi}{2}$ is the period.

For $y = \mathbf{a}\sin \mathbf{b}(x - \mathbf{c}) + \mathbf{d}$ and
$y = \mathbf{a}\cos \mathbf{b}(x - \mathbf{c}) + \mathbf{d}$

Phase Shift

The horizontal shift, c, translates the graph horizontally. Recall (x + c) moves left.

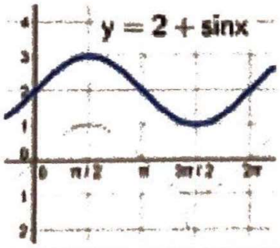




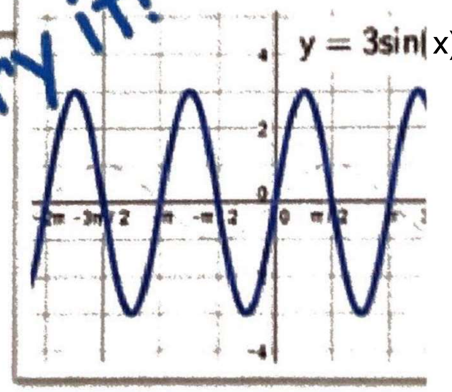
Sine centered about line y = shifted $\frac{\pi}{2}$ to the right

Vertical Shift

The graph oscillates or is centered about the line y = d.



Try it!



Sine stretched 3 above and 3 below the center line y = 0. Two waves in 2π so period is $\frac{2\pi}{2}$ or π radians.

Name:______________________ Date:__________

Sine and Cosine WaVeS

MAX = D + A

MIN = D - A

$y = A\sin B(x - C) + D$

$y = A\cos B(x - C) + D$

|A| = Amplitude

B = Cycle

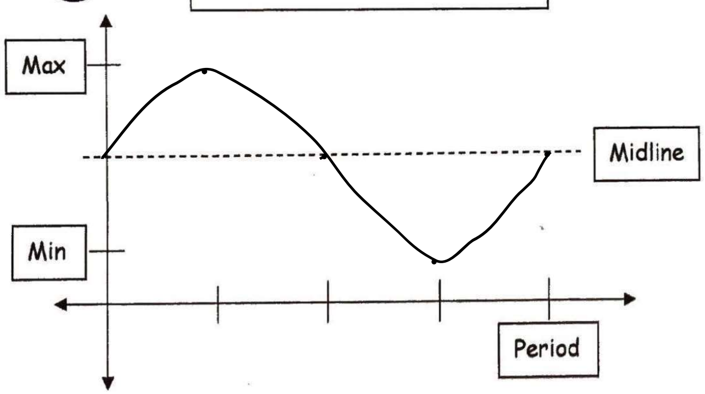
C = Phase Shift

D = Vertical Shift

Period: length of 1 cycle

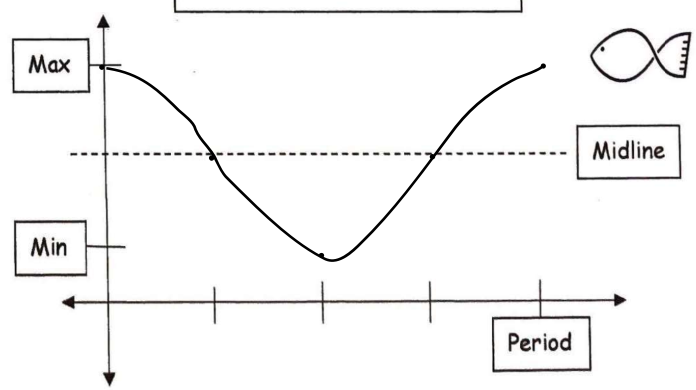
PB = 2π or P = 2pi / B

$y = A\sin B(x - C) + D$



SINE: MIDLINE -> MAX -> MIDLINE -> MIN -> MIDLINE

$y = A\cos B(x - C) + D$

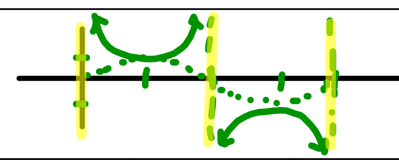
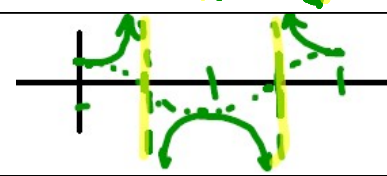
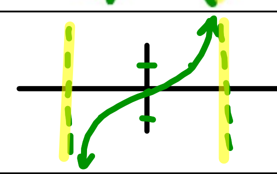
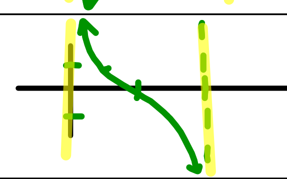


COSINE: MAX -> MIDLINE -> MIN -> MIDLINE -> MAX

PreCalculus - Graphs of the Other Trig Functions

one phase

(bx - c = ?)

function	***period***	***start***	***stop***	***graph***
y=csc(x)	2π	0π	2π	
y=sec(x)	2π	0π	2π	
y=tan(x)	π	-π/2	π/2	
y=cot(x)	π	0π	π	

PreCalculus - Trig Identities

Reciprocal:

$$\sin x = \frac{1}{\csc x} \qquad \csc x = \frac{1}{\sin x}$$

$$\cos x = \frac{1}{\sec x} \qquad \sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{1}{\cot x} \qquad \cot x = \frac{1}{\tan x}$$

Pythagorean:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Cofunctions:

$$\sin\theta = \cos(90° - \theta)$$

$$\cos\theta = \sin(90° - \theta)$$

$$\csc\theta = \sec(90° - \theta)$$

$$\sec\theta = \csc(90° - \theta)$$

$$\tan\theta = \cot(90° - \theta)$$

$$\cot\theta = \tan(90° - \theta)$$

Quotient:

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Even/Odd:

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x \qquad \sec(-x) = \sec x \qquad \cot(-x) = -\cot x$$

Sum/Difference Identities

1. $\sin(A+B)=\sin A\cos B+\cos A\sin B$

2. $\sin(A-B)=\sin A\cos B-\cos A\sin B$

3. $\cos(A+B)=\cos A\cos B-\sin A\sin B$

4. $\cos(A-B)=\cos A\cos B+\sin A\sin B$

5. $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$

6. $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

Half-Angle Identities

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1+\cos A}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1-\cos A}{\sin A}$$

Double-Angle Identities

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\cos(2A) = 1 - 2\sin^2 A$$

$$\cos(2A) = 2\cos^2 A - 1$$

$$\sin(2A) = 2\sin A\cos A$$

$$\tan(2A) = \frac{2\tan A}{1-\tan^2 A}$$

Vectors Cheat Sheet

Vector Magnitude

$\|\langle a, b\rangle\| = \sqrt{a^2 + b^2}$

Unit Vector (Normalizing a Vector)

Divide by the magnitude: $\frac{\vec{u}}{\|\vec{u}\|}$

Adding Vectors

$\langle a, b\rangle + \langle c, d\rangle = \langle a + c, b + d\rangle$

$(a\hat{\imath} + b\hat{\jmath}) + (c\hat{\imath} + d\hat{\jmath}) = (a + c)\hat{\imath} + (b + d)\hat{\jmath}$

Scalar Multiplication

$a\langle b, c\rangle = \langle ab, ac\rangle$

$a(b\hat{\imath} + c\hat{\jmath}) = ab\hat{\imath} + ac\hat{\jmath}$

Dot Products

$\langle a, b\rangle \cdot \langle c, d\rangle = ac + bd$

$(a\hat{\imath} + b\hat{\jmath}) \times (c\hat{\imath} + d\hat{\jmath}) = ac + bd$

Alternative formula: $\vec{u} \cdot \vec{v} = \|\vec{u}\| \, \|\vec{v}\| \cos\theta$

Vector Angles

Finding the direction of a single vector: $\tan\theta = \frac{y}{x}$

Finding the angle separating two vectors: $\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \, \|\vec{v}\|}$

Trig Form:

$$\vec{v} = \left[\!\left[\vec{v}\right]\!\right] \langle \cos\theta, \sin\theta \rangle$$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc \operatorname{Cos} A$$