PreCalculus Name Key

More Sum & Difference Identities WS

1. Using angles from the unit circle, find the EXACT value of $\cos 255^0$.

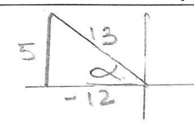
$\cos 255^\circ = \cos(210^\circ + 45^\circ) = \cos 210^\circ \cos 45^\circ - \sin 210^\circ \sin 45^\circ$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{-\sqrt{6}+\sqrt{2}}{4}}$

#2-6. Find the following given: α is in quadrant II and $\csc\alpha = \frac{13}{5}$

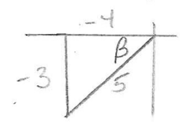
$\sin\alpha = \frac{5}{13}\ \frac{\text{opp}}{\text{hyp}}$



$\cos\alpha = -\frac{12}{13}$

β is in quadrant III and $\cot\beta = \frac{4}{3}$

$\tan\beta = \frac{3}{4}\ \frac{\text{opp}}{\text{adj}}$



$\sin\beta = -\frac{3}{5}$

$\cos\beta = -\frac{4}{5}$

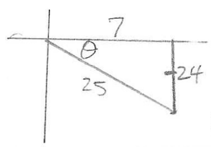
θ is in quadrant IV and $\sec\theta = \frac{25}{7}$

$7^2 + b^2 = 25^2$

$49 + b^2 = 625$

$b^2 = 576$

$b = 24$



$\sin\theta = -\frac{24}{25}$ $\quad \cos\theta = \frac{7}{25}\ \frac{\text{adj}}{\text{hyp}}$

2. $\cos(\beta - \alpha) = \cos\beta\cos\alpha + \sin\beta\sin\alpha$

$= \left(-\frac{4}{5}\right)\left(-\frac{12}{13}\right) + \left(-\frac{3}{5}\right)\left(\frac{5}{13}\right) = \frac{48}{65} - \frac{15}{65} = \boxed{\frac{33}{65}}$

3. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(\frac{5}{13}\right)\left(-\frac{4}{5}\right) + \left(-\frac{12}{13}\right)\left(-\frac{3}{5}\right) = \frac{-20}{65} + \frac{36}{65} = \boxed{\frac{16}{65}}$

4. $\tan(\beta + \theta)$ $\frac{\tan\beta + \tan\theta}{1 - \tan\beta\tan\theta} = \frac{\frac{3}{4} + \frac{-24}{7}}{1 - \left(\frac{3}{4}\right)\left(\frac{-24}{7}\right)} = \frac{\frac{21}{28} - \frac{96}{28}}{1 + \frac{72}{28}} = \frac{\frac{-75}{28}}{\frac{28}{28} + \frac{72}{28}} = \frac{-75}{\cancel{28}} \cdot \frac{\cancel{28}}{100}$

$\boxed{\frac{-3}{4}}$

5. $\sin\left(\theta - \frac{7\pi}{6}\right)$ $\sin\theta\cos\frac{7\pi}{6} - \cos\theta\sin\frac{7\pi}{6} = \left(-\frac{24}{25}\right)\left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{7}{25}\right)\left(-\frac{1}{2}\right)$

$= \frac{24\sqrt{3}}{50} + \frac{7}{50} = \boxed{\frac{24\sqrt{3}+7}{50}}$

6. $\cos\left(\frac{5\pi}{3} + \alpha\right) = \cos\frac{5\pi}{3}\cos\alpha - \sin\frac{5\pi}{3}\sin\alpha$

$= \left(\frac{1}{2}\right)\left(-\frac{12}{13}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{5}{13}\right) = \frac{-12}{26} + \frac{5\sqrt{3}}{26} = \boxed{\frac{-12+5\sqrt{3}}{26}}$

7. Simplify: $\sin\left(\frac{3\pi}{2} + x\right) = \sin\frac{3\pi}{2}\cos x + \cos\frac{3\pi}{2}\sin x$

$= (-1)(\cos x) + \cancel{(0)\sin x} = \boxed{-\cos x}$

#8-10. Solve each of the following equations over the interval $[0, 2\pi)$

8. $\cos\left(\frac{5\pi}{4} - x\right) = \cos\left(\frac{5\pi}{4} + x\right) - 1$

$$\cancel{\cos\frac{5\pi}{4}\cos x} + \sin\frac{5\pi}{4}\sin x = \cancel{\cos\frac{5\pi}{4}\cos x} - \sin\frac{5\pi}{4}\sin x - 1$$

$$+\sin\frac{5\pi}{4}\sin x \qquad\qquad +\sin\frac{5\pi}{4}\sin x$$

$$2\sin\frac{5\pi}{4}\sin x = -1$$

$$2\left(-\frac{\sqrt{2}}{2}\right)\sin x = -1$$

$$-\sqrt{2}\sin x = -1$$

$$\sin x = \frac{1}{\sqrt{2}} \rightarrow \sin x = \frac{\sqrt{2}}{2}$$

$$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}}$$

9. $2\cos\left(x - \frac{3\pi}{2}\right) = \cot\frac{\pi}{6}$

$$2\left(\cos x\cos\frac{3\pi}{2} + \sin x\sin\frac{3\pi}{2}\right) = \sqrt{3}$$

$$2(\cos x(0) + \sin x(-1)) = \sqrt{3}$$

$$2(-\sin x) = \sqrt{3}$$

$$-2\sin x = \sqrt{3}$$

$$\sin x = -\frac{\sqrt{3}}{2}$$

$$\boxed{x = \frac{4\pi}{3}, \frac{5\pi}{3}}$$

10. $\sin\left(x + \frac{7\pi}{2}\right) + 5 - 5\cos x = 8\cos^2 x$

$\frac{7\pi}{2} = \frac{4\pi}{2} + \frac{3\pi}{2} \Rightarrow$ 1 full circle $+ \frac{3\pi}{2}$

$$\sin x\cos\frac{7\pi}{2} + \cos x\sin\frac{7\pi}{2} + 5 - 5\cos x = 8\cos^2 x$$

$$\cancel{\sin x(0)} + \cos x(-1) + 5 - 5\cos x = 8\cos^2 x$$

$$-\cos x + 5 - 5\cos x = 8\cos^2 x$$

$$-6\cos x + 5 = 8\cos^2 x$$

$$0 = 8\cos^2 x + 6\cos x - 5$$

$$(4\cos x + 5)(2\cos x - 1)$$

$$\cancel{\cos x = -\frac{5}{4}} \qquad \cos x = \frac{1}{2} \qquad \boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$$