Name _

As you realize you forgot some concepts, keep a "running" page going of everything you need to re-memorize/learn. Then use it as a reference page for all of your midterm exam studying.

Conics

 Graph and provide the requested information: Circles: Center, Radius
 Ellipses: Center, Vertices, Co-vertices, foci, major and minor axis length Hyperbolas: Center, Vertices, Foci, and Asymptotes
 Parabolas: Vertex, Focus, Directrix, End Points of Latus Rectum

a.
$$(x+1)^2 + (y-3)^2 = 10$$

b. $\frac{(x-2)^2}{9} + \frac{y^2}{25} = 1$
c. $16x^2 - 9y^2 = 144$

d.
$$\frac{(y-2)^2}{25} - \frac{(x+3)^2}{4} = 1$$
 e. $(x+4) + (y-2)^2 = 0$ f. $4(y-1)^2 = 16(x-5)$

- 2. Name the conic and write it in standard form:
 - a. $x^2 + y^2 6x 2y + 1 = 0$
 - b. $6x^2 12 = 6y^2$
 - c. $9x^2 + 4y^2 + 54x 16y + 61 = 0$
 - d. $9x^2 4y^2 + 36x 8y 40 = 0$

e.
$$x^2 + x - y = 5$$

- 3. Write the standard form of the given conic using the given information:
 - a. circle with center (-2, 3) and diameter 8
 - b. horizontal ellipse with center at (3, -4); major axis length 8; minor axis length 4
 - c. circle with center (1, 4) and passes through (2, -1)
 - d. hyperbola with vertices (1, 2) and (5, 2) and the slope of one asymptote is $\frac{3}{2}$
 - e. ellipse with vertices at (2, 1) and (6, 1); co-vertices at (4, 2) and (4, 0)
 - f. hyperbola with vertices $(0,\pm 2)$ and foci $(0,\pm 4)$
 - g. parabola with focus (5, 5), directrix: y = -3
 - h. parabola with vertex (2, -1), passes through (4, 2), p > 0, axis of symmetry: x = 2

4. Solve the systems of equations by graphing.

a.
$$x^2 + y^2 = 16$$
b. $(x + 1)^2 + (y - 3)^2 = 1$ c. $(x + 1)^2 + (y - 1)^2 = 1$ $x - y = 4$ $x^2 + y^2 - 4x - 5 = 0$ $(x - 2)^2 + (y - 1)^2 = 4$

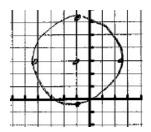
5. Solve the systems algebraically.

a.
$$x^2 + y^2 = 5$$
b. $x^2 + y^2 = 9$ c. $4x^2 + 9y^2 - 36y = 0$ $y = -x + 3$ $x^2 + y^2 - 4x + 3 = 0$ $x^2 + 9y - 27 = 0$

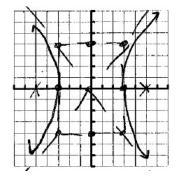
Answers

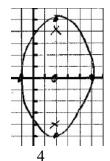
1a) c = (-1,3); r =
$$\sqrt{10} \approx 3.16$$

1b) c = (2,0); v = (2,5), (2,-5); cv = (-1,0), (5,0); f = (2,4), (2,-4); ma = 10; mi = 6



1c) c = (0,0); v = (-3,0), (3,0); f = (-5,0), (5,0); a = $y = \pm \frac{4}{3}x$





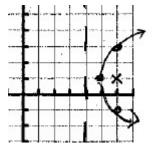
1d)
$$c = (-3,2); v = (-3,-3), (-3,7); f = (-3,2 \pm \sqrt{29})$$

a: $y-2 = \pm \frac{5}{2}(x+3)$

1e)
$$v = (-4,2); f = \left(-\frac{17}{4}, 2\right); d = x = -\frac{15}{4};$$

 $Ir = \left(-\frac{17}{4}, \frac{3}{2}\right), \left(-\frac{17}{4}, \frac{5}{2}\right)$

1f) v = (5,1); f = (6,1); d = x = 4; Ir = (6,3), (6,-1)

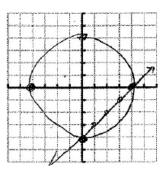


2a) Circle; $(x-3)^2 + (y-1)^2 = 9$ 2b) Hy	perbola; $\frac{x^2}{2} - \frac{y^2}{2} = 1$ 2c) Ellipse; $\frac{(x+3)^2}{4} + \frac{(y-1)^2}{2}$	$\left(\frac{-2}{9}\right)^2 = 1$
2d) Hyperbola; $\frac{(x+2)^2}{8} - \frac{(y+1)^2}{18} = 1$	2e) Parabola; $\left(x + \frac{1}{2}\right)^2 = y + \frac{21}{4}$	
3a) $(x+2)^2 + (y-3)^2 = 16$ 3b) $\frac{(x+2)^2}{(x+2)^2} = 16$	$\frac{-3)^2}{16} + \frac{(y+4)^2}{4} = 1$ 3c) $(x-1)^2 + (y-4)^2$	$(4)^2 = 26$

- $\begin{pmatrix} x+2 \end{pmatrix} + \begin{pmatrix} y & 3 \end{pmatrix}$ 3g) $(x-5)^2 = 16(y-1)$
- $(x-1)^{-} + (y-4)^{-}$ = 26 Soly $(x+2)^2 + (y-3)^2 = 10$ Soly $\frac{16}{16} + \frac{1}{4} = 1$ Soly $(x-1)^2 + (y-1)^2$ Soly $(x-1)^2 + (y-1)^2$ Soly $\frac{y^2}{4} - \frac{x^2}{12} = 1$ Soly $\frac{y^2}{4} - \frac{x^2}{12} = 1$ 3h) $(x-2)^2 = \frac{4}{3}(y+1)$

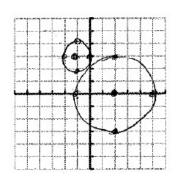
4a) (0,-4), (4,0)

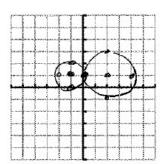
4b) No solution



5a) (2,1), (1,2)

5b) (3,0)





4c) (0,1)

5c) (3,2), (-3,2)

PreCalculus Review for Midterm Exam Matrices

Given the following matrices, simplify the expressions, using fractions instead of decimals.

$$A = \begin{bmatrix} -3 & 2 \\ 0 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -3 \\ 4 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix} \quad D = \begin{bmatrix} -1 & 4 & 0 \\ 3 & -5 & 2 \\ -4 & 3 & -2 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 6 & -3 \\ 2 & -4 & -1 \end{bmatrix}$$
1. 3(A-C)
2. |D|
3. -½(ED)
4. C⁻¹
5. B²
6. |B|
7. 2A - 3B + C
8. Evaluate.

- 3
 4
 -1

 -2
 3
 0

 1
 2
 0
- 9. Solve.

$$\begin{vmatrix} 5 & 7x \\ -x & -6 \end{vmatrix} = -2$$

- 10. Solve for x and y. $2\begin{bmatrix} x+2\\ y-3 \end{bmatrix} + \begin{bmatrix} 5\\ -4 \end{bmatrix} = \begin{bmatrix} 7\\ 1 \end{bmatrix}$
- 11. Solve using a matrix equation. 2x + 4y = -53x - 7y = 4
- 13. Multiply: $\begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 2 & -1 \end{bmatrix}$ 14. Multiply: $\begin{bmatrix} 1 & 5 & -4 \\ 6 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & -3 \\ 1 & 1 \end{bmatrix}$

12. Solve for x and y. $\begin{bmatrix} x & -7 \end{bmatrix} \begin{bmatrix} 2 \end{bmatrix} \begin{bmatrix} 10 \end{bmatrix}$

$$\begin{bmatrix} x & -7 \\ 3 & y \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$$

15. Find the inverse of

a)
$$\begin{bmatrix} 3 & -4 \\ 4 & -2 \end{bmatrix}$$
 b) $\begin{bmatrix} 2 & 4 \\ -6 & -12 \end{bmatrix}$

16. You can only find the inverse of a _____ matrix.

17. If $A_{2x3} \cdot B_{3x1} = C$ find the dimensions of C. _____

Word Problems. (a) define the variables (b) write the system of equations (c) write the matrix representation of the system (d) write your answer in a complete sentence.

18. The perimeter of a rectangular picture is 86 inches. Twice the width exceeds the length by 2 inches. What are the dimensions of the picture?

19. Mrs. Mardis buys 2 granola bars and 3 coffee's for \$21.83. Mrs. Doyle buys 5 granola bars and 1 coffee for \$15.90. How much does one granola bar and one coffee cost?

20. Your team goes to eat at a restaurant. There are 26 people eating dinner. Some team members order the buffet for \$12.99 and some order the grilled steak meal for \$15.95. Coach got the bill. It was \$364.38. How many people ordered the buffet?

21. Ramona spent \$17.00 on two different types of lollipops for Spring Fling prizes. Some cost \$0.50 and some cost \$0.35. If she bought a total of 40 lollipops, how many of each kind did she buy?

22. Flourish and Blotts store sells books. Some cost \$6.00 and some cost \$7.00. On Wednesday, Flourish and Blotts sold 27 books for \$177.00. How many of each did they sell?

23. At a spring concert, tickets for adults cost \$4.00 and tickets for students cost \$2.50. How many of each kind of ticket were purchased if 125 tickets were bought for \$413.

Answers

1.
$$\begin{bmatrix} -27 & 18 \\ -9 & 21 \end{bmatrix}$$
2. -123. $\begin{bmatrix} -29 & 35 & -9 \\ 2 & 2 & 3 \end{bmatrix}$ 4. Not possible5. $\begin{bmatrix} -8 & -3 \\ 4 & -11 \end{bmatrix}$ 6. 107. $\begin{bmatrix} -6 & 9 \\ -9 & 11 \end{bmatrix}$ 8. 79. $x = \pm 2$ 10. $x = -1; y = \frac{11}{2}$ 11. $\left(\frac{-19}{26}, \frac{-23}{26} \right)$ 12. $x = \frac{45}{2}; y = -1$ 13. $\begin{bmatrix} 1 & 19 \\ 4 & -2 \end{bmatrix}$ 14. $\begin{bmatrix} 13 & -20 \\ 11 & -7 \end{bmatrix}$ 15. a) $\begin{bmatrix} -\frac{1}{5} & \frac{2}{5} \\ -\frac{2}{5} & \frac{3}{10} \end{bmatrix}$ b) Not possible16. Square

17. 2 x 1

18. a) L = length, W = width

b)
$$2L + 2W = 86$$

 $-L + 2W = 2$ c) $\begin{bmatrix} 2 & 2\\ -1 & 2 \end{bmatrix} \begin{bmatrix} L\\ W \end{bmatrix} = \begin{bmatrix} 86\\ 2 \end{bmatrix}$

d) The length of the picture is 28 inches and the width is 15 inches.

19. a) g = granola, c = coffee b) $\begin{array}{c} 2g + 3c = 21.83\\ 5g + c = 15.90\end{array}$ c) $\begin{array}{c} 2 & 3\\ 5 & 1 \end{array} \begin{bmatrix} g\\ c \end{bmatrix} = \begin{bmatrix} 21.83\\ 15.9 \end{bmatrix}$

d) The granola costs \$1.99 and the coffee costs \$5.95.

20. a) b= buffet, g = grilled steak b)
$$\begin{array}{c} 12.99b + 15.95g = 364.38\\ b + g = 26 \end{array}$$
 c) $\begin{bmatrix} 12.99 & 15.95\\ 1 & 1 \end{bmatrix} \begin{bmatrix} b\\ g \end{bmatrix} = \begin{bmatrix} 364.38\\ 26 \end{bmatrix}$

d) 17 people ordered the buffet.

21. a) x = lollipop 1, y = lollipop 2 b)
$$\begin{array}{c} .5x + .35y = 17 \\ x + y = 40 \end{array}$$
 c) $\begin{bmatrix} .5 & .35 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 17 \\ 40 \end{bmatrix}$

d) She bought 20 of each kind.

22. a)
$$x = \$6$$
 dollar book, $y = \$7$ book b)
$$\begin{array}{c} 6x + 7y = 177\\ x + y = 27 \end{array}$$
 c)
$$\begin{bmatrix} 6 & 7\\ 1 & 1 \end{bmatrix} \begin{bmatrix} x\\ y \end{bmatrix} = \begin{bmatrix} 177\\ 27 \end{bmatrix}$$

d) They sold 12 of the \$6 books and 15 of the \$17 books

23. a) a = adults, s = students b)
$$\begin{array}{l} 4a + 2.5s = 413 \\ a + s = 125 \end{array}$$
 c)
$$\begin{bmatrix} 4 & 2.5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ s \end{bmatrix} = \begin{bmatrix} 413 \\ 125 \end{bmatrix}$$

d) 67 adults and 58 student tickets were purchased.

PreCalculus Review for Midterm Exam Intro to Trig – Angles & SOHCAHTOA

In which quadrant does θ terminate if:
 (a) sine is negative and tangent is negative
 (b) cosecant is positive and secant is negative
 (c) cotangent is negative and cosine is positive

2. In which quadrant does each angle terminate? (a) $\frac{8\pi}{5}$ (b) -543° (c) $\frac{23\pi}{7}$ (d) 420°

3. Find a positive and negative angle that is coterminal with: (a) 73° (b) $\frac{4\pi}{7}$

- 4. Change each to degree measure: (a) $\frac{8\pi}{3}$ (b) $-\frac{4\pi}{15}$
- 5. Change to radian measure: (a) 125° (b) -540°
- 6. If $\sec x = \frac{8}{3}$ and x is an angle in Quadrant IV, find: (a) $\sin x$ (b) $\cos x$ (c) $\tan x$ (d) $\csc x$ (e) $\cot x$

7. If (-5, -6) lies on the terminal side of angle A in standard position, find:

- (a) $\sin A$ (b) $\cos A$ (c) $\tan A$ (d) $\csc A$ (e) $\sec A$ (f) $\cot A$
- 8. Find the reference angle for each: (a) 117° (b) $\frac{15\pi}{8}$ (c) -290° (d) $-\frac{29\pi}{11}$

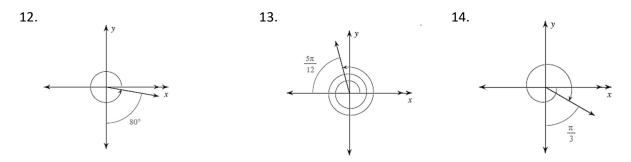
9. For what angles between 0 and 2 $\pi\,$ is the secant function undefined?

10. Find the value of each function:

(a)
$$\cos 720^{\circ}$$
 (b) $\sec 150^{\circ}$ (c) $\sin(-60^{\circ})$ (d) $\tan 225^{\circ}$ (e) $\csc 180^{\circ}$ (f) $\cot(-120^{\circ})$
(g) $\cos \frac{8\pi}{3}$ (h) $\tan \left(-\frac{3\pi}{2}\right)$ (i) $\sec \frac{2\pi}{3}$ (j) $\sin \frac{13\pi}{6}$ (k) $\csc \frac{7\pi}{3}$ (l) $\cot 6\pi$

11. If $\csc\theta > 0$ and $\cot\theta < 0$, in which quadrant does θ terminate?

Find the measure of each angle.



15. Find an angle between 0° and 360° that is coterminal with -435°

16. Find an angle between 0 and 2π that is coterminal with $\frac{11\pi}{3}$.

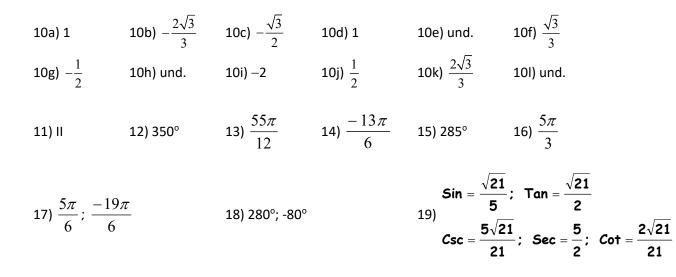
Find a positive and a negative coterminal angle for each given angle.

17.
$$\frac{-7\pi}{6}$$
 18. 640°

19. Cos $\theta = \frac{2}{5}$, where Sin $\theta > 0$ and tan $\theta > 0$. Find the exact values of the five remaining trig functions.

Answers

1a) quadrant IV1b) quadrant II1c) quadrant IV2a) quadrant IV2B) quadrant II2c) quadrant IV2a) quadrant IV2B) quadrant II2c) quadrant III3a) 433°, -287°3b)
$$\frac{18\pi}{7}, \frac{-10\pi}{7}$$
2c) quadrant III4a) 480°4b) -48°5a) $\frac{25\pi}{36}$ 5b) -3\pi6a) $-\frac{\sqrt{55}}{8}$ 6b) $\frac{3}{8}$ 6c) $-\frac{\sqrt{55}}{3}$ 6d) $-\frac{8\sqrt{55}}{55}$ 6e) $-\frac{3\sqrt{55}}{55}$ 7a) $-\frac{6\sqrt{61}}{61}$ 7b) $-\frac{5\sqrt{61}}{61}$ 7c) $\frac{6}{5}$ 7d) $-\frac{\sqrt{61}}{6}$ 7e) $-\frac{\sqrt{61}}{5}$ 7f) $\frac{5}{6}$ 8a) $\theta' = 63^\circ$ 8b) $\theta' = \frac{\pi}{8}$ 8c) $\theta' = 70^\circ$ 8d) $\theta' = \frac{4\pi}{11}$ 9) $\frac{\pi}{2}, \frac{3\pi}{2}$



PreCalculus Review for Midterm Exam Triangle Trig – Right Triangles & Laws of Sines and Cosines

- 1. Solve triangle ABC if angle B is a right angle, side a = 11.1 and side b = 14.7.
- 2. Solve triangle ABC is angle A = 42.3° , side b = 6.1 and side c = 8.3.
- 3. Find the area of triangle ABC in question #2.
- 4. Solve triangle ABC if angle A = 82.1° , angle B = 38.7° and side a = 12.9.
- 5. Solve triangle ABC if side a = 9, side b = 11 and side c = 14.
- 6. Find the area of triangle ABC in question #5.

7. Find the angle of elevation to the top of a 10.3 foot tree that is casting a 28.9 foot shadow.

8. A ship leaves port and sails with a bearing of S 42° W at a speed of 23 mph. After 5 hours, how far south and how far west is the ship from the port?

9. At Pope High School, there is a flagpole mounted on the roof. From a point 200 feet in front of Pope, the angles of elevation to the base of the flagpole on the roof and to the top of the flagpole on the roof are 24° and 37° respectfully. Find the height of the flagpole.

10. From the top of a 55 foot tall lighthouse, a Coast Guard officer sights a boat in difficulty. The angle of depression to the boat is 6° . How far is the boat from the shoreline?

11. A 70 foot line is attached to a kite. When the kite has pulled the line tight, the angle of elevation of the kite is 56° . Find the height of the kite.

12. A plane is 250 miles north and 175 miles east of the airport. What bearing should the pilot follow if he wants to fly directly to the airport?

Solve the triangles using the Law of Sines, Law of Cosines or both.

- 13. a = 8.5 b = 12 $A = 42^{\circ}$
- 14. $C = 120^{\circ}$ a = 4 b = 6
- 15. a = 55 b = 34 c = 70
- 16. $C = 15^{\circ}$ a = 6.25 b = 2.15

Use a calculator to evaluate each function. Round your answers to 4 decimal places.

17. sin 25°18. cos 65°19. cot 71.5°20. sec 42°
--

21. cos 8°50'25"	22. tan	$\frac{\pi}{16}$	23. csc 1.25	24. csc 0			
Find the value of Θ in degrees. Round to the nearest hundredth.							
25. sin θ = 0.8191	26. cos	i Θ = 0.9848	27. tan θ = 1.1920	28. sec Θ = 1.4123			
Find the value of Θ in D°M'. Round to the nearest minute.							
29. cos θ = 0.4223	30. tan	Θ = 1.5002	31. csc θ = 1.5555	32. cot Θ = 2.1234			
Answers							
1) A = 49°, C = 41°, c = 9	9.6	2) B = 51.8° C = 85.9°, a = 5.6		3) 17.0 square units			
4) C = 59.2°, b = 8.1, c = 11.2 5) option 1: A = 39.9°, B = 51.8°, C = 88.3° option 2: A = 40°, B = 50.8°, C = 89.2°		6) 49.5 square units					
7) 19.6° 8		8) south 85.5 miles, west 77 miles		9) 61.7 feet			
10) 523.3 feet		11) 58 feet		12) W 55° S or S 35° W			
13) B = 70.8°, C = 67.2°, c = 11.7 AND B = 109.2°, C = 28.8°, c = 6.1		14) options 1: c = 8.7, A = 23.5°, B = 36.5° option 2: c = 8.7, A = 23.3°, B = 36.7°					
15) C = 101.1°, A = 50.4, B = 28.5° 16) A = 157.4°, B = 7.6°, C		, c = 4.2	17) 0.4226				
18) 0.4226	19) 0.3346	20) 1.3499	21) 0.9881	22) 0.1989			
23) 1.0538	24) undefined	25) 54.99°	26) 10°	27) 50.01°			
28) 44.92°	29) 65°1'	30) 56°19'	31) 40°0'	32) 25°13′			

PreCalculus Review for Midterm Exam Graphs of Trig Functions & Sinusoidal Models

- 1. Given: The domain of a sinusoid is $[-82^{\circ}, 998^{\circ}]$
 - (a) The phase shift of the function is _____ (b) The period of the function is _____
- 2. Given: The range of a sinusoidal function is [-24, -7]. Find the following:
 - (a) amplitude of the function _____ (b) vertical shift of the function _____

Find the amplitude, period, vertical shift, and phase shift of the following.

3.
$$y = 3\sin\left(\frac{x}{4} + \frac{\pi}{12}\right) - 1$$

4. $y = -2\cos\left(3\theta - 120^\circ\right) + 2$

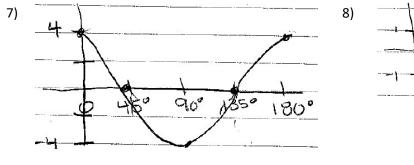
Graph each of the following.

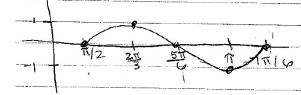
5. $y = -2\sin\left(\frac{1}{3}x + \frac{\pi}{6}\right) + 1$ 6. $y = 3\cos 2(\theta - 60^{\circ}) - 2$ 7. $y = 4\cos(2\theta)$ 8. $y = \sin\left(3x - \frac{3\pi}{2}\right)$ 9. $y = -2\cos\left(\frac{1}{2}x + \frac{5\pi}{6}\right) - 2$ 10. $y = 3\sin(3\theta - 45^{\circ})$

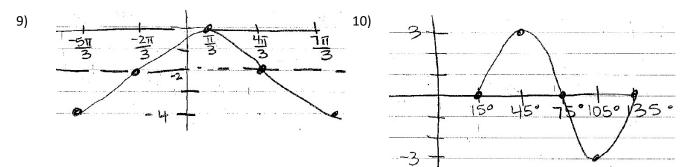
Write the equation of each function.

- 11. Write the equation of a cosine function that has amplitude 5, period 270° , phase shift 60° , and vertical shift 3.
- 12. Write the equation of a sine graph with amplitude 2, period π , phase shift $-\frac{\pi}{4}$, and vertical shift -4.
- 13. Write the equation of a sine function whose domain is $[8^0, 68^0]$ and whose range is [4, 7].

Answers 1a) -82° 1b) 1080° 2a) 8.5 2b) -15.5 $ps = -\frac{\pi}{3}$ 3) amp = 3period = 8π vs = -1 ١ period = 120° 4) amp = 2 $ps = 40^{\circ}$ vs = 2 5) 6) 600 150° 195 05° 51 411 2 E







11) $y = 5\cos\frac{4}{3}(\theta - 60^\circ) + 3$ 12) $y = 2\sin 2\left(x + \frac{\pi}{4}\right) - 4$ 13) $y = 1.5\sin 6(\theta - 8^\circ) + 5.5$