Trigonometric Equations Maze!

Directions: Solve each equation on the given interval. Use your solutions to navigate through the maze. **Staple all work to this paper!**

Start!

1 $\sin 2x = -\sqrt{3}\cos x$ $\left[\pi, \frac{3\pi}{2}\right]$

$\left\{\frac{4\pi}{3}, \frac{3\pi}{2}\right\}$

2 $\tan\frac{x}{2} + \cot x = \sqrt{2}$ $\left[\frac{\pi}{2}, 2\pi\right)$

$\left\{\frac{7\pi}{4}\right\}$

$\cos x - \sin\frac{x}{2} = 0$ $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

$\left\{\pi, \frac{7\pi}{6}\right\}$ $\left\{\frac{3\pi}{4}\right\}$ $\left\{\frac{5\pi}{4}\right\}$ $\left\{\frac{3\pi}{4}, \frac{5\pi}{4}\right\}$ $\left\{\frac{5\pi}{6}, \pi\right\}$

3 $\tan 2x + \tan x = 0$ $\left[\frac{\pi}{2}, \pi\right]$

$\left\{\frac{5\pi}{6}, \pi\right\}$

6 $1 - \sin x = \cos 2x$ $\left[0, \frac{\pi}{2}\right]$

$\left\{0, \frac{\pi}{6}\right\}$

7 $\sin\left(x - \frac{\pi}{2}\right) + \sin 2x = 0$ $\left[\frac{\pi}{2}, \pi\right]$

$\left\{\frac{2\pi}{3}, \pi\right\}$ $\left\{\frac{\pi}{2}, \frac{2\pi}{3}\right\}$ $\left\{\pi, \frac{5\pi}{3}\right\}$ $\left\{\frac{2\pi}{3}, \pi\right\}$ $\left\{\frac{\pi}{2}, \frac{5\pi}{6}\right\}$

4 $\sin\left(x - \frac{\pi}{3}\right) = \cos\left(x - \frac{\pi}{6}\right) - \frac{\sqrt{6}}{2}$ $\left[0, \frac{3\pi}{2}\right]$

$\left\{\frac{\pi}{4}\right\}$

5 $4\cos^2\frac{x}{2} - 2\sin^2 x = \cos x + 1$ $[\pi, 2\pi)$

$\left\{\frac{\pi}{3}, \pi\right\}$

8 $\tan\left(x - \frac{\pi}{4}\right) \cdot \sec^2 x = 0$ $[\pi, 2\pi)$

$\left\{\frac{5\pi}{4}\right\}$ $\left\{\frac{\pi}{6}, \pi\right\}$ $\left\{\frac{\pi}{3}, \frac{\pi}{2}\right\}$ $\left\{\frac{\pi}{6}, \frac{\pi}{2}\right\}$ $\left\{\frac{5\pi}{4}\right\}$

14 $\cos x - \cos\frac{x}{2} = 0$ $\left[\pi, \frac{3\pi}{2}\right]$

$\left\{\frac{4\pi}{3}\right\}$

End! ☺

$\left\{\frac{7\pi}{4}\right\}$

9 $\sin(x + \pi) - 2\cos^2 x = 2\cos\left(x - \frac{\pi}{2}\right)$ $\left[\frac{3\pi}{2}, 2\pi\right)$

$\left\{\frac{7\pi}{6}\right\}$ $\left\{\frac{3\pi}{2}, \frac{7\pi}{4}\right\}$ $\left\{\frac{7\pi}{4}, 2\pi\right\}$ $\left\{\frac{5\pi}{3}\right\}$ $\left\{\frac{11\pi}{6}\right\}$

$4\sin^2 x + \cos 2x = 3\sin x$ $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

$\left\{\frac{3\pi}{2}, \frac{5\pi}{3}\right\}$

13 $\sqrt{2}\cos x + \sin 2x = 0$ $\left[\frac{3\pi}{2}, 2\pi\right)$

$\left\{\pi, \frac{4\pi}{3}\right\}$

10 $\cos 2x + \cos x = 0$ $[\pi, 2\pi)$

$\left\{\frac{\pi}{2}, \frac{7\pi}{6}\right\}$ $\left\{\frac{11\pi}{6}, 2\pi\right\}$ $\left\{\frac{5\pi}{4}, \frac{7\pi}{4}\right\}$ $\left\{\frac{5\pi}{6}\right\}$ $\left\{\pi, \frac{5\pi}{3}\right\}$

$\tan\left(x - \frac{\pi}{3}\right) = \tan 2x$ $[0, 2\pi)$

$\left\{\frac{7\pi}{4}\right\}$

12 $6\sin^2 x + \cos 2x - 3 = 0$ $[\pi, 2\pi)$

$\left\{\frac{2\pi}{3}\right\}$

11 $2\sin^2\frac{x}{2} - 4\cos^2 x = -\cos x$ $\left[\frac{\pi}{2}, \pi\right]$

Trig Equations Maze

Start:

Interval

(1) $\sin 2x = -\sqrt{3}\cos x$ $\quad [\pi, \frac{3\pi}{2}]$

$2\sin x\cos x + \sqrt{3}\cos x = 0$

$\cos x(2\sin x + \sqrt{3}) = 0$

$\cos x = 0 \qquad 2\sin x + \sqrt{3} = 0$

$2\sin x = -\sqrt{3}$

$\sin x = -\sqrt{3}/2$

within interval

$x = \cancel{\frac{\pi}{2}}, \boxed{\frac{3\pi}{2}, \frac{4\pi}{3}} \cancel{\frac{5\pi}{3}}$

(2) $\tan\frac{x}{2} + \cot x = \sqrt{2}$ $\quad [\frac{\pi}{2}, 2\pi)$

$\frac{1-\cos x}{\sin x} + \frac{\cos x}{\sin x} = \sqrt{2}$

$\frac{1 - \cancel{\cos x} + \cancel{\cos x}}{\sin x} = \sqrt{2}$

$\frac{1}{\sin x} = \sqrt{2}$

$\csc x = \sqrt{2}$

$\sin x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\sin x = \frac{\sqrt{2}}{2}$

within interval

$x = \cancel{\frac{\pi}{4}}, \boxed{\frac{3\pi}{4}}$

(3) $\tan 2x + \tan x = 0$ $\quad [\frac{\pi}{2}, \pi]$

$(1-\tan^2 x)\left(\frac{2\tan x}{1-\tan^2 x} + \tan x = 0\right)$

$2\tan x + \tan x(1-\tan^2 x) = 0$

$2\tan x + \tan x - \tan^3 x = 0$

$\tan^3 x - 3\tan x = 0$

$\tan x(\tan^2 x - 3) = 0$

$\tan x = 0 \qquad \tan^2 x - 3 = 0$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$x = \cancel{0\pi}, \boxed{\pi} \qquad x = \cancel{\frac{\pi}{3}}, \boxed{\frac{2\pi}{3}}, \cancel{\frac{4\pi}{3}}, \cancel{\frac{5\pi}{3}}$

4) $\overset{A}{\sin}\left(x - \overset{B}{\frac{\pi}{3}}\right) = \cos\left(\overset{A}{x} - \overset{B}{\frac{\pi}{6}}\right) - \frac{\sqrt{6}}{2}$ Interval $\left[0, \frac{3\pi}{2}\right]$

$$\sin x \cos\frac{\pi}{3} - \cos x \sin\frac{\pi}{3} = \cos x \cos\frac{\pi}{6} + \sin x \sin\frac{\pi}{6} - \frac{\sqrt{6}}{2}$$

$$\cancel{\frac{1}{2}\sin x} - \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{3}}{2}\cos x + \cancel{\frac{1}{2}\sin x} - \frac{\sqrt{6}}{2}$$

$$-\frac{\sqrt{3}}{2}\cos x \qquad -\frac{\sqrt{3}}{2}\cos x$$

$$-\frac{2\sqrt{3}}{2}\cos x = -\frac{\sqrt{6}}{2}$$

$$\frac{-\sqrt{3}\cos x}{-\sqrt{3}} = \frac{\frac{-\sqrt{6}}{2}}{-\sqrt{3}}$$

$$\cos x = \frac{-\sqrt{6}}{2} \cdot \frac{-1}{\sqrt{3}} = \frac{\sqrt{6}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{18}}{6} = \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2}$$

$$\cos x = \frac{\sqrt{2}}{2} \quad , x = \boxed{\frac{\pi}{4}}, \cancel{\frac{2\pi}{4}}$$

5. $4\cos^2\frac{x}{2} - 2\sin^2 x = \cos x + 1$ $[\pi, 2\pi)$

$$4\left(\sqrt{\frac{1+\cos x}{2}}\right)^2 - 2\sin^2 x = \cos x + 1$$

$$\cancel{4}^{2}\left(\frac{1+\cos x}{\cancel{2}_{1}}\right) - 2\sin^2 x = \cos x + 1$$

$$2 + 2\cos x - 2(1-\cos^2 x) = \cos x + 1$$

$$2 + 2\cos x - 2 + 2\cos^2 x = \cos x + 1$$

$$-1 \quad -\cos x \qquad -\cos x - 1$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$$2\cos x - 1 = 0 \qquad \cos x + 1 = 0$$

$$\cos x = -1$$

$$2\cos x = 1$$

$$\cos x = \frac{1}{2} \qquad x = \boxed{\pi}$$

$$x = \cancel{\frac{\pi}{3}}, \boxed{\frac{5\pi}{3}}$$

(6) $1 - \sin x = \cos 2x$ $\quad [0, \frac{\pi}{2}]$

$$1 - \sin x = 1 - 2\sin^2 x$$
$$-1 + 2\sin^2 x \quad -1 + 2\sin^2 x$$
$$2\sin^2 x - \sin x = 0$$

$$\sin x(2\sin x - 1) = 0$$

$\sin x = 0$ $\qquad$ $2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = 1/2$

$x = \boxed{0\pi}, \cancel{\pi}$ $\qquad$ $x = \boxed{\frac{\pi}{6}}, \cancel{\frac{5\pi}{6}}$

(7) $\sin(\overset{A}{x} - \overset{B}{\frac{\pi}{2}}) + \sin 2x = 0$ $\quad [\frac{\pi}{2}, \pi]$

$$\sin x \cos\frac{\pi}{2} - \cos x \sin\frac{\pi}{2} + 2\sin x\cos x = 0$$
$$\cancel{\sin x(0)} - \cos x(1) + 2\sin x\cos x = 0$$
$$-\cos x + 2\sin x\cos x = 0$$
$$\cos x(-1 + 2\sin x) = 0$$

$\cos x = 0$ $\qquad$ $-1 + 2\sin x = 0$

$x = \boxed{\frac{\pi}{2}}, \cancel{\frac{3\pi}{2}}$ $\qquad$ $2\sin x = 1$

$\sin x = 1/2$

$x = \cancel{\frac{\pi}{6}}, \boxed{\frac{5\pi}{6}}$

(8) $\tan(\overset{A}{x} - \overset{B}{\frac{\pi}{4}}) \cdot \sec^2 x = 0$ $\quad [\pi, 2\pi)$

$$\frac{\tan x - \tan\frac{\pi}{4}}{1 + \tan x\tan\frac{\pi}{4}} \cdot \sec^2 x = 0$$

$$\left(\frac{\tan x - 1}{1 + \tan x(1)}\right)(\sec^2 x) = 0$$

$\frac{\tan x - 1}{1 + \tan x} = 0$ $\qquad$ $\sec^2 x = 0$

$\sec x = 0$

$\tan x - 1 = 0$ $\qquad$ $\cos x = \emptyset$

$\tan x = 1$ $\quad$ $x = \cancel{\frac{\pi}{4}}, \boxed{\frac{5\pi}{4}}$ $\qquad$ Not Possible

(9) $\sin(x+\pi) - 2\cos^2 x = 2\cos\left(x - \frac{\pi}{2}\right)$ $\left[\frac{3\pi}{2}, 2\pi\right)$

$\sin x \underset{(-1)}{\cos\pi} + \cancel{\cos x \underset{(0)}{\sin\pi}} - 2\cos^2 x = 2\left(\cancel{\cos x \underset{(0)}{\cos\frac{\pi}{2}}} + \sin x \underset{(1)}{\sin\frac{\pi}{2}}\right)$

$-\sin x - 2\cos^2 x = 2\sin x$

$-\sin x - 2(1 - \sin^2 x) = 2\sin x$

$-\sin x - 2 + 2\sin^2 x - 2\sin x = 0$

$2\sin^2 x - 3\sin x - 2 = 0$

$(2\sin x + 1)(\sin x - 2) = 0$

$2\sin x + 1 = 0$ $\quad$ $\sin x - 2 = 0$

$2\sin x = -1$ $\quad$ $\sin x = \cancel{2}$

$\sin x = -1/2$ $\quad$ Not Possible

$x = \cancel{\frac{7\pi}{6}}, \boxed{\frac{11\pi}{6}}$

(10) $\cos 2x + \cos x = 0$ $\quad$ $[\pi, 2\pi)$

$2\cos^2 x - 1 + \cos x = 0$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$ $\quad$ $\cos x + 1 = 0$

$2\cos x = 1$ $\quad$ $\cos x = -1$

$\cos x = 1/2$ $\quad$ $x = \pi$

$x = \cancel{\frac{\pi}{3}}, \boxed{\frac{5\pi}{3}}$

(11) $2\sin^2\frac{x}{2} - 4\cos^2 x = -\cos x$ $\quad$ $\left[\frac{\pi}{2}, \pi\right]$

$2\left(\pm\sqrt{\frac{1-\cos x}{2}}\right)^2 - 4\cos^2 x = -\cos x$

$\cancel{2}\left(\frac{1-\cos x}{\cancel{2}}\right) - 4\cos^2 x = -\cos x$

$1 - \cancel{\cos x} - 4\cos^2 x + \cancel{\cos x} = 0$

$-4\cos^2 x + 1 = 0$

$-4\cos^2 x = -1$

$\cos^2 x = \frac{1}{4}$ $\quad$ $x = \cancel{\frac{\pi}{3}}, \boxed{\frac{2\pi}{3}}, \cancel{\frac{4\pi}{3}}, \cancel{\frac{5\pi}{3}}$

$\cos x = \pm\frac{1}{2}$

(12) $6\sin^2 x + \cos 2x - 3 = 0$ $[\pi, 2\pi)$

$6\sin^2 x + 1 - 2\sin^2 x - 3 = 0$

$4\sin^2 x - 2 = 0$

$\frac{4\sin^2 x = 2}{4}$

$\sqrt{\sin^2 x} = \sqrt{\frac{1}{2}}$

$\sin x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$x = \cancel{\frac{\pi}{4}}, \cancel{\frac{3\pi}{4}}, \boxed{\frac{5\pi}{4}, \frac{7\pi}{4}}$

(13) $\sqrt{2}\cos x + \sin 2x = 0$ $[\frac{3\pi}{2}, 2\pi)$

$\sqrt{2}\cos x + 2\sin x\cos x = 0$

$\cos x(\sqrt{2} + 2\sin x) = 0$

$\cos x = 0$ $\sqrt{2} + 2\sin x = 0$

$2\sin x = -\sqrt{2}$

$x = \cancel{\frac{\pi}{2}}, \boxed{\frac{3\pi}{2}}$ $\sin x = -\frac{\sqrt{2}}{2}$

$x = \cancel{\frac{5\pi}{4}}, \boxed{\frac{7\pi}{4}}$

(14) $\cos x - \cos\frac{x}{2} = 0$ $[\pi, \frac{3\pi}{2}]$

$\cos x - \pm\sqrt{\frac{1+\cos x}{2}} = 0$

$(\cos x)^2 = \left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2$

$(2)\cos^2 x = \frac{1+\cos x}{\cancel{2}}(\cancel{2})$

$2\cos^2 x = 1 + \cos x$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0$ $\cos x - 1 = 0$

$2\cos x = -1$ $\cos x = 1$

$\cos x = -\frac{1}{2}$ $x = \cancel{0\pi}$

$x = \cancel{\frac{2\pi}{3}}, \boxed{\frac{4\pi}{3}}$