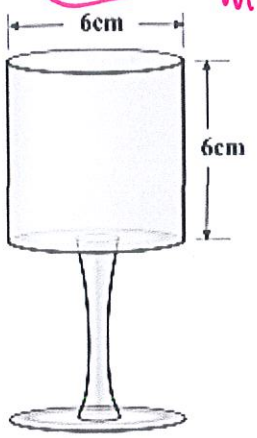
Geometry
Cross Sections & Cavalieri's Principle

Name: key
Date: _____

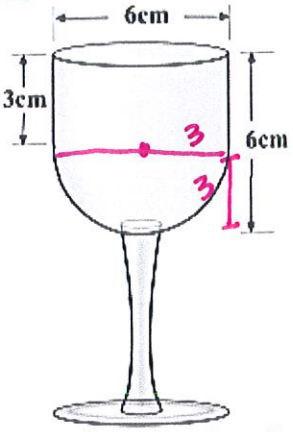
WARM-UP: Find the volume of each glass. Which of the following holds the most liquid?

Glass 1 *holds the most liquid.



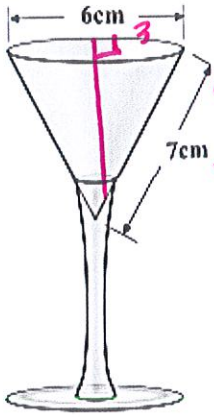
$V = \pi 3^2 \cdot 6$
$= 169.6\ cm^3$

Glass 2



$V_{cyl} = \pi 3^2 \cdot 3 = 84.8\ cm^3$
$V_{hemi} = \frac{4}{3}\pi 3^3 = 113.1\ cm^3$
$\frac{113.1}{2} = 56.6$
$V_{Total} = 84.8 + 56.6 = 141.4\ cm^3$

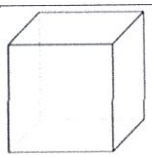
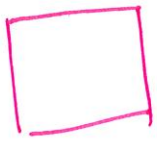
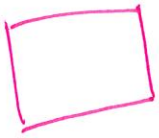
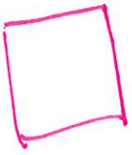
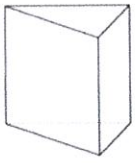
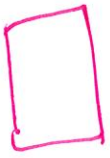
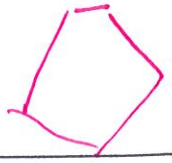
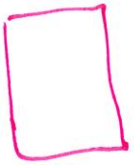
Glass 3



$3^2 + b^2 = 7^2$
$9 + b^2 = 49$
$\sqrt{b^2} = \sqrt{40}$
$b \approx 6.3$

$V = \frac{1}{3}\pi 3^2 \cdot 6.3$
$= 59.4\ cm^3$

CROSS SECTIONS – The shape you get when cutting straight through a 3-dimensional shape.

	NAME	SHAPE OF BASE	Which figure results when you slice it PARALLEL to the base?	Which figure results with you slice it PERPENDICULAR to the base?	Which figure results when you slice it DIAGONAL to the base?
	rectangular prism	rectangle			
	triangular prism	Triangle			
	cylinder	circle			

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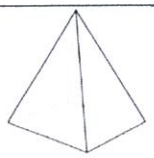
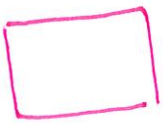
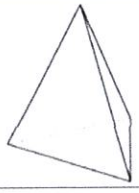
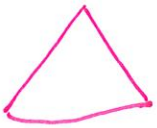
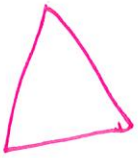
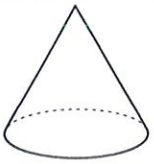
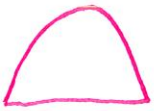
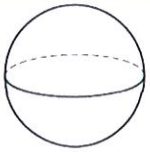
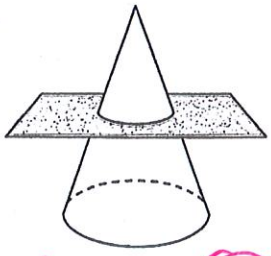
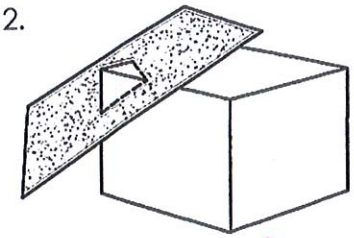
	rectangular pyramid	rectangle			
	triangular pyramid	triangle			
	cone	circle			
	sphere	X			

PRACTICE

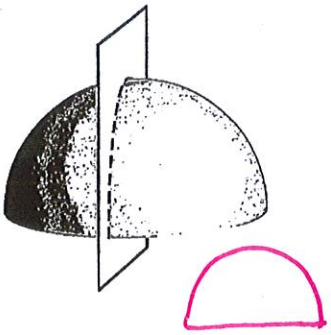
What are the shapes of the cross sections below?

1. 

circle 

2. 

triangle 

3. 

4. Andrew had a piece of foam in the shape of a rectangular prism as shown below. The base is a square with sides 3 inches long, and the piece is 5 inches tall. He cut the foam along the diagonal plane shown by the shaded area. What is the area of the shaded diagonal plane?

$3^2+5^2=c^2$

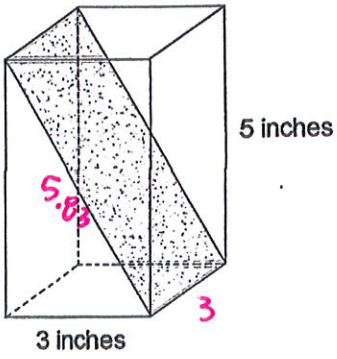
$9+25=c^2$

$\sqrt{34}=c^2$

$c \approx 5.83$

$A_{\square} = 5.83 \cdot 3$

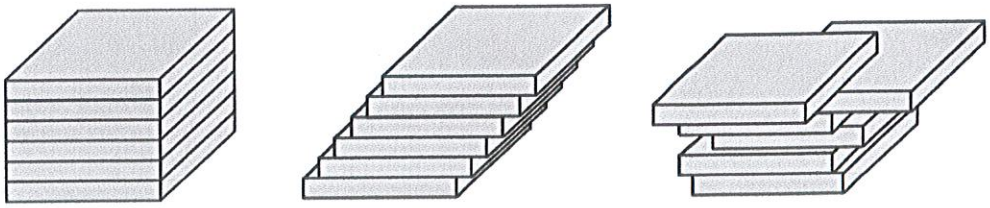
$= 17.49 \text{ in}^2$



Bonaventura Cavalieri – He was an Italian mathematician born in 1598 – 1657. He is known for his work in optics, motion, calculus, and introduction of logarithms.

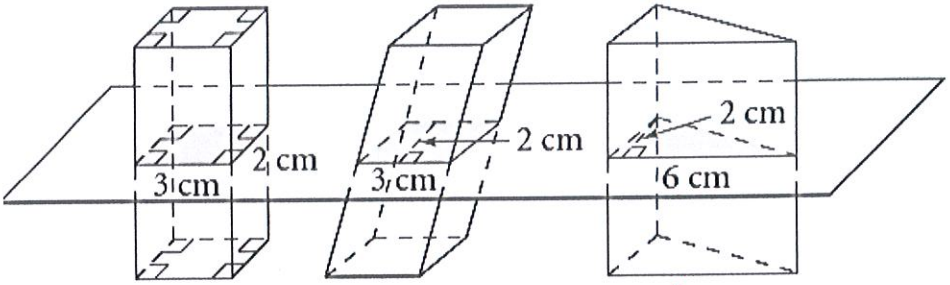
CAVALIERI'S PRINCIPLE

- The volumes of two objects of the same height are equal if the areas of their corresponding cross sections are equal.



These pieces maintain the **SAME** volume regardless of how they are moved!!

Find the area of each cross section below.

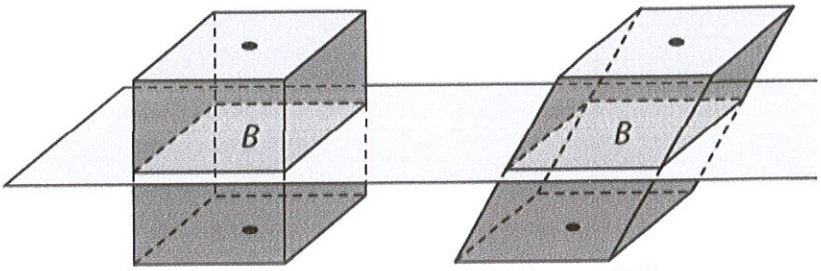


$A = 3 \cdot 2 = 6\text{cm}^2$ $A = 3 \cdot 2 = 6\text{cm}^2$ $A = \frac{1}{2} \cdot 6 \cdot 2 = 6\text{cm}^2$

Based on what we know about the areas of the cross sections, what can we assume about the volumes (assuming all three heights are the same)?

Since the areas of all their cross sections are congruent, then the volumes of all 3 figures are congruent.

The same volume formula applies whether it's a **right** prism or an **oblique** prism.

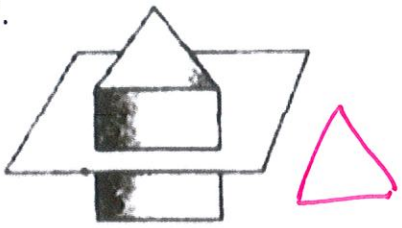


right prism

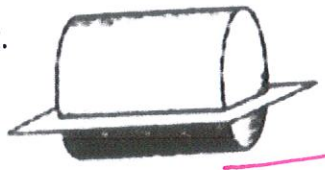
oblique prism

PRACTICE:

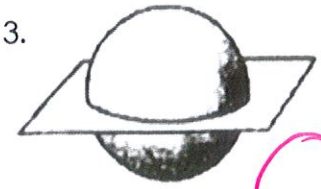
Name the cross section.

1. 

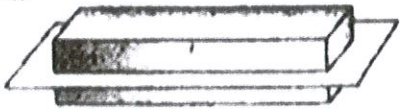
triangle

2. 

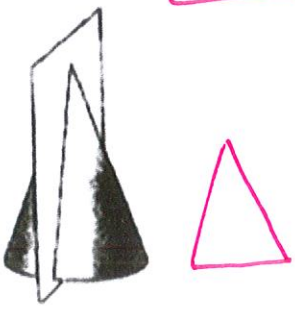
rectangle

3. 

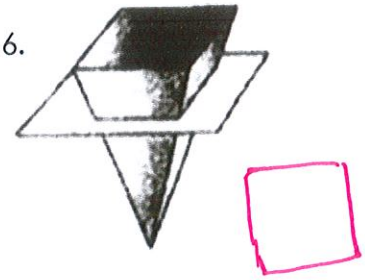
circle

4. 

rectangle

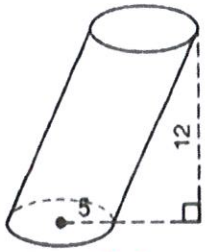
5. 

triangle

6. 

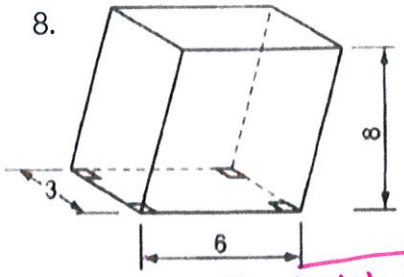
square

Find the volume of each oblique figure.

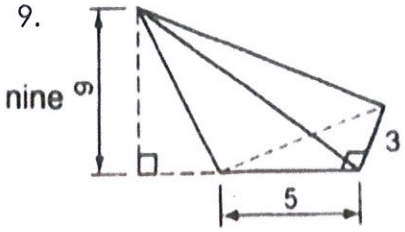
7. 

$V = \pi 5^2 \cdot 12$

$= 942.5 u^3$

8. 

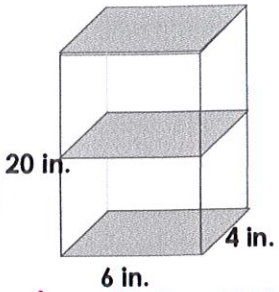
$V = 3 \cdot 6 \cdot 8 = 144 u^3$

9. 

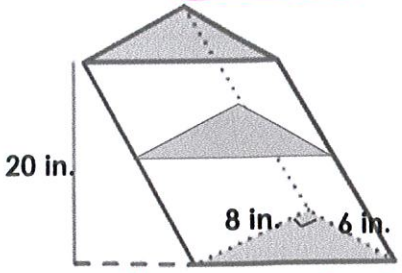
$V = \frac{1}{3}\left(\frac{1}{2} \cdot 5 \cdot 3\right) \cdot 9$

$= 22.5 u^3$

10. Which figure has more volume?



$A_{\square} = 4 \cdot 6 = 24 in^2$



$A_{\triangle} = \frac{1}{2} \cdot 8 \cdot 6 = 24 in^2$

Since the areas of the bases / cross sections are congruent and height are congruent, then volumes are congruent, by Cavalieri's!!

11. Collin is going to change the oil in his Jeep. He has two funnels. A has a diameter of 6 inches and is 5 inches deep. B has a diameter of 5 inches but is 7 inches deep. He wants to use the funnel with the greatest volume to minimize the chance of spilling the oil. What are the volumes of the funnels? Which one should he use A or B?

The areas of the bases are congruent, but the heights are different.

$A \rightarrow V = \pi 3^2 \cdot 5 = 141.4$

$B \rightarrow V = \pi 2.5^2 \cdot 7 = 137.4$

Use A!!