Solving Trig Equations with Sum + Diff. Identities

1. $\tan(x+\pi) + 2\sin(x+\pi) = 0$

$$\frac{\tan x + \cancel{\tan \pi}^{0}}{1 - \tan x \cancel{\tan \pi}_{0}} + 2\left(\sin x \cancel{\cos \pi}^{(-1)} + \cos x \cancel{\sin \pi}^{(0)}\right) = 0$$

$$\tan x + 2(-\sin x) = 0$$

$$\tan x - 2\sin x = 0$$

$$\cos x\left(\frac{\sin x}{\cos x} - 2\sin x = 0\right)$$

$$\sin x - 2\sin x \cos x = 0$$

$$\sin x(1 - 2\cos x) = 0$$

$$\sin x = 0 \qquad 1 - 2\cos x = 0$$

$$-2\cos x = -1$$

$$\cos x = \frac{1}{2}$$

$$\boxed{x = 0\pi, \pi, \frac{\pi}{3}, \frac{5\pi}{3}}$$

2. $\sin\left(x+\frac{\pi}{6}\right) - \sin\left(x-\frac{\pi}{6}\right) = \frac{1}{2}$

$$\sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6} - \left(\sin x \cos\frac{\pi}{6} - \cos x \sin\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\cancel{\sin x \cos\frac{\pi}{6}} + \cos x \sin\frac{\pi}{6} - \cancel{\sin x \cos\frac{\pi}{6}} + \cos x \sin\frac{\pi}{6} = \frac{1}{2}$$

$$\left(\frac{1}{2}\right) 2\cos x \sin\frac{\pi}{6} = \frac{1}{2}\left(\frac{1}{2}\right)$$

$$\cos x \sin\frac{\pi}{6} = \frac{1}{4}$$

$$(2)\ \cos x\left(\frac{1}{2}\right) = \frac{1}{4}\ (2)$$

$$\cos x = \frac{1}{2} \qquad \boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$$

3. $\sin\left(x+\frac{\pi}{2}\right) - \cos\left(x+\frac{3\pi}{2}\right) = 0$

$$\sin x \cos\frac{\pi}{2} + \cos x \sin\frac{\pi}{2} - \left(\cos x \cos\frac{3\pi}{2} - \sin x \sin\frac{3\pi}{2}\right) = 0$$

$$\sin x \cos\frac{\pi}{2} + \cos x \sin\frac{\pi}{2} - \cos x \cos\frac{3\pi}{2} + \sin x \sin\frac{3\pi}{2} = 0$$

$$\cancel{\sin x (0)} + \cos x(1) - \cancel{\cos x (0)} + \sin x(-1) = 0$$

$$\cos x - \sin x = 0$$

$$\cos x = \sin x \qquad \boxed{x = \cancel{\frac{\pi}{4}}, \frac{5\pi}{4}}$$

outside of interval $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right)$

4. $-\cos x = 1 + 2\cos(x-\pi)$

$-\cos x = 1 + 2(\cos x \overset{(-1)}{\cancel{\cos\pi}} + \sin x \overset{(0)}{\cancel{\sin\pi}})$

$-\cos x = 1 - 2\cos x$

$\cos x = 1 \longrightarrow \boxed{x = 0\pi}$

5. $4\sin(x+\pi) = 2\cos(x+\frac{\pi}{2}) + 2$

$4(\sin x \overset{(-1)}{\cancel{\cos\pi}} + \cos x \overset{(0)}{\cancel{\sin\pi}}) = 2(\cos x \overset{(0)}{\cancel{\cos\frac{\pi}{2}}} - \sin x \overset{(1)}{\cancel{\sin\frac{\pi}{2}}}) + 2$

$-4\sin x = -2\sin x + 2$

$-2\sin x = 2$

$\sin x = -1 \longrightarrow \boxed{x = \frac{3\pi}{2}}$

6. $\tan(-105°) = \tan(45° - 150°)$

$= \frac{\tan 45° - \tan 150°}{1 + \tan 45° \tan 150°} = \frac{1 - (-\frac{\sqrt{3}}{3})}{1 + (1)(-\frac{\sqrt{3}}{3})} = \frac{\frac{3}{3} + \frac{\sqrt{3}}{3}}{\frac{3}{3} - \frac{\sqrt{3}}{3}} = \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}}$

$= \frac{3+\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3-\sqrt{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}} \cdot \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{9+6\sqrt{3}+3}{9-3} = \frac{12+6\sqrt{3}}{6} = \boxed{2+\sqrt{3}}$

7. $\tan(x+\pi) + \cos(x+\frac{\pi}{2}) = 0$

$\frac{\tan x + \overset{(0)}{\cancel{\tan\pi}}}{1 - \tan x \underset{(0)}{\cancel{\tan\pi}}} + \cos x \overset{(0)}{\cancel{\cos\frac{\pi}{2}}} - \sin x \overset{(1)}{\cancel{\sin\frac{\pi}{2}}} = 0$

$\tan x - \sin x = 0$

$\tan x = \sin x$ $\boxed{x = 0\pi, \pi}$

8. $\cos(x+\frac{\pi}{3}) + \cos(x-\frac{\pi}{3}) = \cos^2 x$

$\cos x \cos\frac{\pi}{3} - \cancel{\sin x \sin\frac{\pi}{3}} + \cos x \cos\frac{\pi}{3} + \cancel{\sin x \sin\frac{\pi}{3}} = \cos^2 x$

$2\cos x \overset{(\frac{1}{2})}{\cancel{\cos\frac{\pi}{3}}} = \cos^2 x$

$\cos x = \cos^2 x$

$0 = \cos^2 x - \cos x$

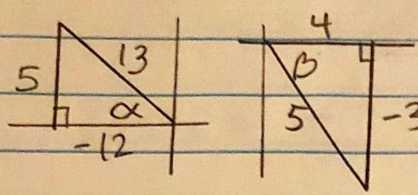
$0 = \cos x(\cos x - 1)$

$\cos x = 0$ $\quad \cos x - 1 = 0$

$\cos x = 1$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}, 0\pi}$

9. $\cos(\beta+\alpha) = \cos\beta\cos\alpha - \sin\beta\sin\alpha$



$\left(\frac{4}{5}\right)\left(\frac{-12}{13}\right) - \left(\frac{-3}{5}\right)\left(\frac{5}{13}\right)$

$\frac{-48}{65} + \frac{15}{65} = \boxed{\frac{-33}{65}}$

$\csc\alpha = \frac{13}{5}$ $\quad \tan\beta = -\frac{3}{4}\,\frac{o}{a}$

$\sin\alpha = \frac{5}{13}\,\frac{o}{h}$

10. $\cos\left(x+\frac{\pi}{6}\right) - \frac{1}{2} = \cos\left(x-\frac{\pi}{6}\right)$

$\cancel{\cos x\cos\frac{\pi}{6}} - \sin x\sin\frac{\pi}{6} - \frac{1}{2} = \cancel{\cos x\cos\frac{\pi}{6}} + \sin x\sin\frac{\pi}{6}$

$+\left(\frac{1}{2}\right) \quad -2\sin x\sin\frac{\pi}{6} = \frac{1}{2} \quad +\left(\frac{1}{2}\right)$

$\sin x\sin\frac{\pi}{6} = -\frac{1}{4}$

$(2) \quad \sin x\left(\frac{1}{2}\right) = -\frac{1}{4} \quad (2)$

$\sin x = -\frac{1}{2} \longrightarrow \boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

11. $\tan(\pi+x) + \tan(\pi+x) = 2$

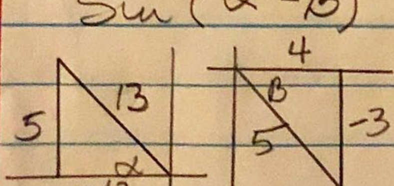
$\frac{\cancel{\tan\pi}^{(0)} + \tan x}{1 - \cancel{\tan\pi}_{(0)}\tan x} + \frac{\cancel{\tan\pi}^{(0)} + \tan x}{1 - \cancel{\tan\pi}_{(0)}\tan x} = 2$

$\tan x + \tan x = 2$

$2\tan x = 2$

$\tan x = 1 \longrightarrow \boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

12. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$



$\left(\frac{5}{13}\right)\left(\frac{4}{5}\right) - \left(\frac{-12}{13}\right)\left(\frac{-3}{5}\right)$

$\frac{20}{65} - \frac{36}{65} = \boxed{\frac{-16}{65}}$

$\csc\alpha = \frac{13}{5}$ $\quad \tan\beta = -\frac{3}{4}\,\frac{o}{a}$

$\sin\alpha = \frac{5}{13}\,\frac{o}{h}$

$$\sin\left(\frac{\pi}{2}-x\right)=\frac{1}{2}$$

$$\cancel{\sin\frac{\pi}{2}}^{(1)}\cos x-\cancel{\cos\frac{\pi}{2}}^{(0)}\sin x=\frac{1}{2}$$

$$\cos x=\frac{1}{2}\longrightarrow\boxed{x=\frac{\pi}{3},\ \cancel{\frac{5\pi}{3}}}$$

outside of $\left(0\pi,\frac{\pi}{2}\right)$

$$3\cos\left(x-\frac{\pi}{2}\right)+1=-2\sin^2 x$$

$$3\left(\cos x\,\cancel{\cos\frac{\pi}{2}}^{(0)}+\sin x\,\cancel{\sin\frac{\pi}{2}}^{(1)}\right)+1=-2\sin^2 x$$

$$3\sin x+1=-2\sin^2 x$$

$$2\sin^2 x+3\sin x+1=0$$

$$(2\sin x+1)(\sin x+1)=0$$

$$2\sin x+1=0 \qquad \sin x+1=0$$

$$2\sin x=-1 \qquad \sin x=-1$$

$$\sin x=-\frac{1}{2}$$

$$\boxed{x=\frac{7\pi}{6},\ \cancel{\frac{11\pi}{6}},\ \frac{3\pi}{2}}$$

outside of $\left(\frac{\pi}{2},\frac{3\pi}{2}\right]$