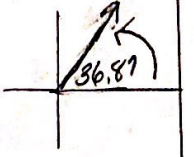
Unit Vectors WS
2020

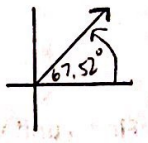
Name Key

Write each of the following as the sum of unit vectors; also, find the magnitude and the direction of the vector. Round your answers to the nearest hundredth.

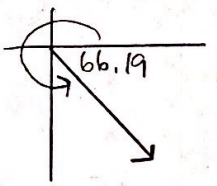
1. Initial point: $(-3,-5)$; Terminal point: $(5,1)$

Vector as sum of unit vectors:	Magnitude:	Direction:
$\langle 5-{-3}, 1-{-5}\rangle$ $\langle 8, 6\rangle$ $\boxed{8\vec{i}+6\vec{j}}$	$\sqrt{8^2+6^2}$ $=\sqrt{64+36}$ $=\sqrt{100}$ $=\boxed{10}$	$\theta' = \tan^{-1}\left(\frac{6}{8}\right)$ $\theta' = \tan^{-1}\left(\frac{3}{4}\right)$ $\boxed{\theta = 36.87^\circ}$ 

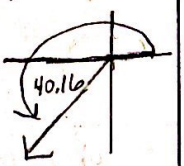
2. Initial point: $(-3,11)$; Terminal point: $(9,40)$

Vector as sum of unit vectors:	Magnitude:	Direction:
$\langle 9-{-3}, 40-11\rangle$ $\langle 12, 29\rangle$ $\boxed{12\vec{i}+29\vec{j}}$	$\sqrt{12^2+29^2}$ $=\sqrt{144+841}$ $=\sqrt{985}$ $=\boxed{31.38}$	$\theta' = \tan^{-1}\left(\frac{29}{12}\right)$ $\boxed{\theta' = 67.52^\circ}$ 

3. Initial point: $(-4.2,5)$; Terminal point: $(3.7,-12.9)$

Vector as sum of unit vectors:	Magnitude:	Direction:
$\langle 3.7-{-4.2}, -12.9-5\rangle$ $\langle 7.9, -17.9\rangle$ $\boxed{7.9\vec{i}-17.9\vec{j}}$	$\sqrt{7.9^2+(-17.9)^2}$ $=\sqrt{382.82}$ $=\boxed{19.57}$ 	$\theta' = \tan^{-1}\left(\frac{-17.9}{7.9}\right)$ $\theta' = -66.19$ $360-66.19=\boxed{293.81^\circ}$

4. Initial point: $(1.64,7.21)$; Terminal point: $(-2.33,3.86)$

Vector as sum of unit vectors:	Magnitude:	Direction:
$\langle -2.33-1.64, 3.86-7.21\rangle$ $\langle -3.97, -3.35\rangle$ $\boxed{-3.97\vec{i}-3.35\vec{j}}$	$\sqrt{(-3.97)^2+(-3.35)^2}$ $=\sqrt{26.9834}$ $=\boxed{5.19}$ 	$\theta' = \tan^{-1}\left(\frac{-3.35}{-3.97}\right)$ $\theta' = \tan^{-1}\left(\frac{3.35}{3.97}\right)$ $\theta' = 40.16$ $\theta = 180+40.16$ $\boxed{\theta = 220.16}$

Answers:

1) $8\vec{i}+6\vec{j}$; $\|\vec{v}\|=10$; $\theta=36.87^\circ$

2) $12\vec{i}+29\vec{j}$; $\|\vec{v}\|=31.38$; $\theta=67.52^\circ$

3) $7.9\vec{i}-17.9\vec{j}$; $\|\vec{v}\|=19.57$; $\theta=293.81^\circ$

4) $-3.97\vec{i}-3.35\vec{j}$; $\|\vec{v}\|=5.19$; $\theta=220.16^\circ$

For each of the following, find:

(a) $-5\vec{u}+2\vec{v}$

(b) $\frac{1}{2}\vec{u}-\vec{v}$

Write answers in the form of the original vectors.

5. $\vec{u}=\langle 5,3\rangle$, $\vec{v}=\langle -4,0\rangle$

a) $-5\langle 5,3\rangle+2\langle -4,0\rangle$

$=\langle -25,-15\rangle+\langle -8,0\rangle=\boxed{\langle -33,-15\rangle}$

b) $\frac{1}{2}\langle 5,3\rangle-\langle -4,0\rangle$

$=\langle \frac{5}{2},\frac{3}{2}\rangle+\langle \frac{8}{2},0\rangle=\boxed{\langle \frac{13}{2},\frac{3}{2}\rangle}$

6. $\vec{u}=\vec{i}+\vec{j}$, $\vec{v}=2\vec{i}-3\vec{j}$ $\vec{u}=\langle 1,1\rangle$ $\vec{v}=\langle 2,-3\rangle$

a) $-5\langle 1,1\rangle+2\langle 2,-3\rangle$

$=\langle -5,-5\rangle+\langle 4,-6\rangle=\langle -1,-11\rangle=\boxed{-\vec{i}-11\vec{j}}$

b) $\frac{1}{2}\langle 1,1\rangle-\langle 2,-3\rangle$

$=\langle \frac{1}{2},\frac{1}{2}\rangle+\langle -\frac{4}{2},\frac{6}{2}\rangle=\langle -\frac{3}{2},\frac{7}{2}\rangle=\boxed{-\frac{3}{2}\vec{i}+\frac{7}{2}\vec{j}}$

7. $\vec{u}=-9\vec{j}$, $\vec{v}=-6\vec{i}+10\vec{j}$

$\vec{u}=\langle 0,-9\rangle$ $\vec{v}=\langle -6,10\rangle$

a) $-5\langle 0,-9\rangle+2\langle -6,10\rangle$

$=\langle 0,45\rangle+\langle -12,20\rangle=\langle -12,65\rangle=\boxed{-12\vec{i}+65\vec{j}}$

b) $\frac{1}{2}\langle 0,-9\rangle-\langle -6,10\rangle$

$=\langle 0,-\frac{9}{2}\rangle+\langle 6,-\frac{20}{2}\rangle=\langle 6,-\frac{29}{2}\rangle=\boxed{6\vec{i}-\frac{29}{2}\vec{j}}$

8. $\vec{u}=\langle 2,-1\rangle$, $\vec{v}=\langle -1,1\rangle$

a) $-5\langle 2,-1\rangle+2\langle -1,1\rangle$

$=\langle -10,5\rangle+\langle -2,2\rangle=\boxed{\langle -12,7\rangle}$

b) $\frac{1}{2}\langle 2,-1\rangle-\langle -1,1\rangle$

$=\langle 1,-\frac{1}{2}\rangle+\langle 1,-\frac{2}{2}\rangle=\boxed{\langle 2,-\frac{3}{2}\rangle}$

Find a unit vector in the direction of the given vector. Write your answer in the same form as the original vector.

9. $\vec{v}=\langle 6,0\rangle$

$\|\vec{v}\|=\sqrt{6^2+0^2}=\sqrt{36}=6$

$\vec{u}=\frac{\langle 6,0\rangle}{6}=\frac{1}{6}\langle 6,0\rangle$

$=\boxed{\langle 1,0\rangle}$

10. $\vec{v}=\langle -4,4\rangle$

$\|\vec{v}\|=\sqrt{(-4)^2+4^2}=\sqrt{32}=4\sqrt{2}$

$\vec{u}=\frac{\langle -4,4\rangle}{4\sqrt{2}}=\frac{1}{4\sqrt{2}}\langle -4,4\rangle$

$=\langle \frac{-4}{4\sqrt{2}},\frac{4}{4\sqrt{2}}\rangle=\langle -\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\rangle$

$=\boxed{\langle -\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\rangle}$

11. $\vec{t}=\langle 5,-12\rangle$

$\|\vec{t}\|=\sqrt{5^2+(-12)^2}=\sqrt{169}=13$

$\vec{u}=\frac{\langle 5,-12\rangle}{13}=\frac{1}{13}\langle 5,-12\rangle$

$=\boxed{\langle \frac{5}{13},-\frac{12}{13}\rangle}$

12. $\vec{m}=4\vec{i}-3\vec{j}$ $\langle 4,-3\rangle$

$\|\vec{m}\|=\sqrt{4^2+(-3)^2}=\sqrt{25}=5$

$\vec{u}=\frac{\langle 4,-3\rangle}{5}=\frac{1}{5}\langle 4,-3\rangle$

$=\langle \frac{4}{5},-\frac{3}{5}\rangle=\boxed{\frac{4}{5}\vec{i}-\frac{3}{5}\vec{j}}$

13. $\vec{w}=\vec{i}-2\vec{j}$ $\langle 1,-2\rangle$

$\|\vec{w}\|=\sqrt{1^2+(-2)^2}=\sqrt{5}$

$\vec{u}=\frac{\langle 1,-2\rangle}{\sqrt{5}}=\frac{1}{\sqrt{5}}\langle 1,-2\rangle$

$=\langle \frac{1}{\sqrt{5}},-\frac{2}{\sqrt{5}}\rangle=\langle \frac{\sqrt{5}}{5},-\frac{2\sqrt{5}}{5}\rangle$

$=\boxed{\frac{\sqrt{5}}{5}\vec{i}-\frac{2\sqrt{5}}{5}\vec{j}}$

14. $\vec{w}=-3\vec{i}$ $\langle -3,0\rangle$

$\|\vec{w}\|=\sqrt{(-3)^2+(0)^2}=\sqrt{9}=3$

$\vec{u}=\frac{\langle -3,0\rangle}{3}=\frac{1}{3}\langle -3,0\rangle$

$=\langle -1,0\rangle=\boxed{-\vec{i}}$

---

5) a) $\langle -33,-15\rangle$ b) $\langle 6.5,1.5\rangle$ 6) a) $-\vec{i}-11\vec{j}$ b) $-1.5\vec{i}+3.5\vec{j}$

7) a) $-12\vec{i}+65\vec{j}$ b) $6\vec{i}-14.5\vec{j}$ 8) a) $\langle -12,7\rangle$ b) $\langle 2,-1.5\rangle$

9) $\langle 1,0\rangle$ 10) $\left\langle -\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right\rangle$ 11) $\left\langle \frac{5}{13},-\frac{12}{13}\right\rangle$ 12) $\frac{4}{5}\vec{i}-\frac{3}{5}\vec{j}$ 13) $\frac{\sqrt{5}}{5}\vec{i}-\frac{2\sqrt{5}}{5}\vec{j}$ 14) $-\vec{i}$