

Solve the following equations over  $[0, 2\pi)$ .

1.  $\csc^2 x + 2 \csc x = 0$

$\csc x (\csc x + 2) = 0$

$\csc x = 0$      $\csc x + 2 = 0$

$\sin x = \emptyset$      $\csc x = -2$

↳ Not Possible     $\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

3.  $2 \cos x \sin x = \cos x$

$2 \cos x \sin x - \cos x = 0$

$\cos x (2 \sin x - 1) = 0$

$\cos x = 0$      $2 \sin x - 1 = 0$

$2 \sin x = 1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

5.  $2 \sin^2 x + 5 \sin x = 3$

$2 \sin^2 x + 5 \sin x - 3 = 0$

$(2 \sin x - 1)(\sin x + 3) = 0$

$2 \sin x = 1$      $\sin x = -3$

$\sin x = \frac{1}{2}$     ↳ not possible

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

7.  $\sin x + \sin x \tan x = 0$

$\sin x (1 + \tan x) = 0$

$\sin x = 0$      $1 + \tan x = 0$

$\tan x = -1$

$x = 0, \pi$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

9.  $\cos^2 x - \cot x \cos^2 x = 0$

$\cos^2 x (1 - \cot x) = 0$

$\cos^2 x = 0$      $1 - \cot x = 0$

$\cos x = 0$      $\cot x = 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

11.  $4 \cos^4 x - 5 \cos^2 x + 1 = 0$

$(4 \cos^2 x - 1)(\cos^2 x - 1) = 0$

$4 \cos^2 x = 1$      $\cos^2 x = 1$

$\cos^2 x = \frac{1}{4}$      $\cos x = \pm 1$

$\cos x = \pm \frac{1}{2}$      $x = 0, \pi$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

2.  $\cos^2 x - \cos x - 2 = 0$

$(\cos x - 2)(\cos x + 1) = 0$

$\cos x - 2 = 0$      $\cos x + 1 = 0$

$\cos x = 2$      $\cos x = -1$

↳ Not Possible

$x = \pi$

4.  $2 = \sec x + \sec^2 x$

$\sec^2 x + \sec x - 2 = 0$

$(\sec x + 2)(\sec x - 1) = 0$

$\sec x = -2$      $\sec x = 1$

$\cos x = -\frac{1}{2}$

$\cos x = 1$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$x = 0$

6.  $\tan^2 x \sin x = \sin x$

$\tan^2 x \sin x - \sin x = 0$

$\sin x (\tan^2 x - 1) = 0$

$\sin x = 0$      $\tan^2 x = 1$

$x = 0, \pi$

$\tan x = \pm 1$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

8.  $\sec x \sin x - 2 \sin x = 0$

$\sin x (\sec x - 2) = 0$

$\sin x = 0$      $\sec x = 2$

$x = 0, \pi$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

~~10.  $3 \csc^2 x + 4 \csc x - 4 = 0$~~

skip

12.  $4 \sin^4 x + \sin^2 x - 3 = 0$

$(4 \sin^2 x - 3)(\sin^2 x + 1) = 0$

$4 \sin^2 x = 3$

$\sin^2 x = -1$

$\sin^2 x = \frac{3}{4}$

$\sin x = \sqrt{-1}$

$\sin x = \pm \frac{\sqrt{3}}{2}$

$\sin x = \pm i$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

↳ imaginary is not possible