Verifying Identities WS 2

1. $\sin^3\theta + \sin\theta\cos^2\theta = \sin\theta$

$\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta$

$\sin\theta\,(1)$

$\sin\theta$ ✓

2. $\dfrac{1+\sec(-x)}{\sin(-x)+\tan(-x)} = -\csc x$

$\dfrac{1+\sec x}{-\sin x - \tan x}$

move neg. $\dfrac{1+\sec x}{-(\sin x + \tan x)}$

$\dfrac{-(1+\sec x)}{\sin x + \tan x}$

$\dfrac{-(1+\sec x)}{\sin x + \frac{\sin x}{\cos x}}$

$\dfrac{-(1+\sec x)}{\sin x\left(1+\frac{1}{\cos x}\right)}$

$\dfrac{-(1+\sec x)}{\sin x\,(1+\sec x)}$

$-\dfrac{1}{\sin x}$

✓

3. $\sec x + \tan x = \frac{\cos x}{1-\sin x} \quad \frac{(1+\sin x)}{(1+\sin x)}$

mult. by conjugate →

$$\frac{\cos x(1+\sin x)}{1-\sin^2 x}$$

$$\frac{\cos x(1+\sin x)}{\cos^2 x}$$

$$\frac{1+\sin x}{\cos x}$$

$$\frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$\sec x + \tan x$ ✓

4. $\frac{\cos x - \cos y}{\sin x + \sin y} + \frac{\sin x - \sin y}{\cos x + \cos y} = 0$

mult. by $\cos x + \cos y$ → ← mult. by $\sin x + \sin y$

$$\frac{(\cos x - \cos y)(\cos x + \cos y) + (\sin x - \sin y)(\sin x + \sin y)}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$\frac{\cos^2 x - \cos^2 y + \sin^2 x - \sin^2 y}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$\frac{\cos^2 x + \sin^2 x - \cos^2 y - \sin^2 y}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$\frac{1 - (\cos^2 y + \sin^2 y)}{(\sin x + \sin y)(\cos x + \cos y)}$$

5. $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

$1 - \sin^2 x - \sin^2 x$

$1 - 2\sin^2 x$ ✓

6. $\dfrac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$

$\dfrac{\cot^2 x}{\csc^2 x}$

$\dfrac{\cos^2 x / \sin^2 x}{1/\sin x}$

$\dfrac{\cos^2 x}{\sin^2 x} \cdot \dfrac{\sin x}{1}$

$\cos^2 x$ ✓

7. $\dfrac{1}{1-\cos x} + \dfrac{1}{1+\cos x} = 2\csc^2 x$

mult. by $1+\cos x$ → ← mult. by $1-\cos x$

$\dfrac{1}{1-\cos x}\dfrac{(1+\cos x)}{(1+\cos x)} + \dfrac{1}{1+\cos x}\dfrac{(1-\cos x)}{(1-\cos x)}$

$\dfrac{1+\cos x + 1 - \cos x}{(1-\cos x)(1+\cos x)}$

$\dfrac{2}{1-\cos^2 x}$

$\dfrac{2}{}$

8. $(\cot^2\theta + 1)(\sin^2\theta - 1) = -\cot^2\theta$

$\csc^2\theta \cdot -\cos^2\theta$

$\frac{1}{\sin^2\theta} \cdot -\cos^2\theta$

$-\frac{\cos^2\theta}{\sin^2\theta}$

$-\cot^2\theta$ ✓

9. $\csc x + \cot x = \frac{\sin x}{1-\cos x}$ ← multi by $1+\cos x$

$\frac{\sin x(1+\cos x)}{(1-\cos x)(1+\cos x)}$

$\frac{\sin x(1+\cos x)}{1-\cos^2 x}$

$\frac{\sin x(1+\cos x)}{\sin^2 x}$

$\frac{1+\cos x}{\sin x}$

$\frac{1}{\sin x} + \frac{\cos x}{\sin x}$

$\csc x + \cot x$ ✓

10. $\dfrac{\sin x\cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y} = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

$$= \frac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{1 - \frac{\sin x}{\cos x} \cdot \frac{\sin y}{\cos y}}$$

mult. by $\cos y$ → ; ← mult. by $\cos x$

← mult. by $\cos x \cos y$

$$= \frac{\frac{\sin x \cos y + \sin y \cos x}{\cos x \cos y}}{\frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y}}$$

$$\frac{\sin x \cos y + \sin y \cos x}{\cos x \cos y} \cdot \frac{\cos x \cos y}{\cos x \cos y - \sin x \sin y}$$

$$\frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$$ ✓