Geometry DAY 8.3
Special Segment Lengths in a Circle

Name: Key **Date:** _____

WARM-UP Solve. Factoring may be required.

1. ~~$x^2 - 4x + 24 = 0$~~

$x^2 - 5x - 24 = 0$

$(x+3)(x-8) = 0$ Factors

$x+3=0 \quad x-8=0$

$\boxed{x=-3} \quad \boxed{x=8}$ Solutions

-24 / -5

24: 1 24, 2 12, 3 8, 4 6

2. $x^2 + 8 = 48$ Isolate x^2

$\sqrt{x^2} = \sqrt{40}$ Take square root of both sides

$x = \pm\sqrt{40} = \pm\sqrt{4}\sqrt{10}$

$\boxed{x = \pm 2\sqrt{10}}$

3. $x^2 - 18x - 88 = 0$

$(x+4)(x-22) = 0$

$x+4=0 \quad x-22=0$

$\boxed{x=-4} \quad \boxed{x=22}$

-88 / -18

88: 1 88, 2 44, 4 22, 8 11

4. $2x^2 + 4 = x^2 + x + 60$ Combine like terms

$-x^2 \quad -x^2$

$x^2 + 4 = x + 60$

$x^2 - x + 4 = 60$

$x^2 - x - 56 = 0$ Factor

$(x+7)(x-8) = 0$

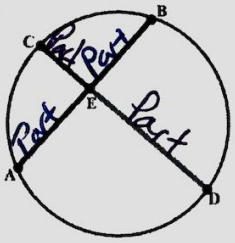
$x+7=0 \quad x-8=0$

$\boxed{x=-7, 8}$

56: 1 56, 2 28, 4 14, 7 8

Two Chords Intersecting

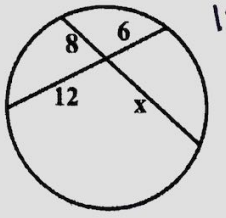
If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$\frac{AE}{\text{Part}} \cdot \frac{EB}{\text{Part}} = \frac{CE}{\text{Part}} \cdot \frac{ED}{\text{Part}}$

Examples:

1.

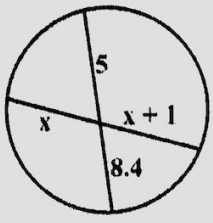


$12 \cdot 6 = 8 \cdot x$

$72 = 8x$

$\boxed{x=9}$

2.



$x(x+1) = 5(8.4)$

$x^2 + x = 42$

$x^2 + x - 42 = 0$

$(x-6)(x+7) = 0$

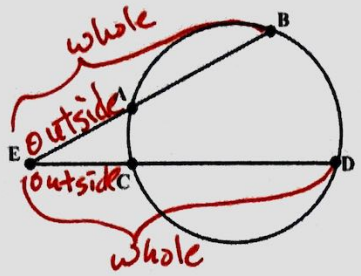
$x-6=0 \quad x+7=0$

$\boxed{x=6} \quad x=-7$

Does not work. Length is positive.

Two Secant Segments Intersecting Outside a Circle

The product of the length of one ~~external~~ outside secant segment and its ~~entire~~ whole segment equals the product of the length of the other ~~external~~ outside secant segment and its ~~entire~~ whole segment.



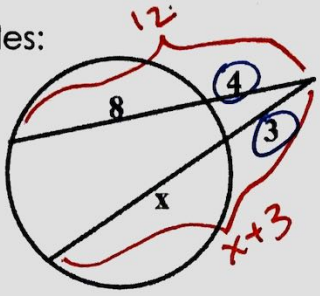
$EA \cdot EB = EC \cdot ED$

outside · whole = outside · whole

ow = ow

Examples:

1.



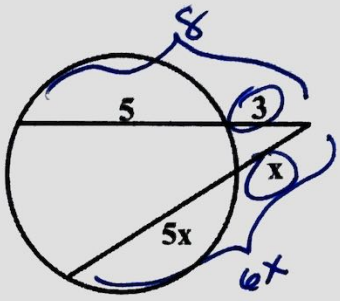
$4 \cdot 12 = 3(x+3)$

$48 = 3x + 9$

$39 = 3x$

$\boxed{x = 13}$

2.



$3 \cdot 8 = x \cdot 6x$

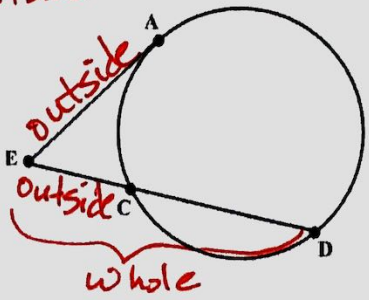
$24 = 6x^2$

$\sqrt{4} = \sqrt{x^2}$

$x = 2$

A Secant Segment and a Tangent Segment Intersecting Outside a Circle

The square of the length of the tangent segment equals the product of the length of the ~~external~~ outside secant segment and the ~~entire~~ whole secant segment.

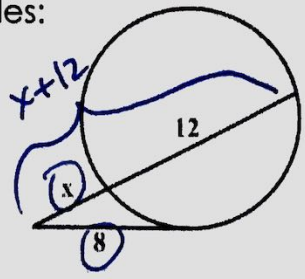


$(EA)^2 = EC \cdot ED$

$(\text{outside})^2 = \text{outside} \cdot \text{whole}$

Examples:

1.



$8^2 = x(x+12)$

$64 = x^2 + 12x$

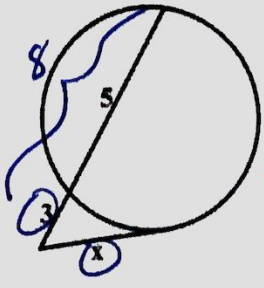
$0 = x^2 + 12x - 64$

$0 = (x+16)(x-4)$

$x+16=0 \quad x-4=0$

$x = -16$ ~~-16~~ $\quad \boxed{x = 4}$

2.



$x^2 = 3 \cdot 8$

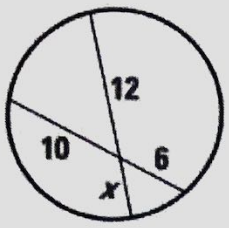
$\sqrt{x^2} = \sqrt{24}$

$x = \sqrt{4}\sqrt{6}$

$\boxed{x = 2\sqrt{6}}$

PUTTING IT ALL TOGETHER – Solve for x.

1.

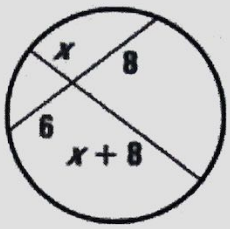


$12 \cdot x = 10 \cdot 6$

$12x = 60$

$\boxed{x = 5}$

2.



$6 \cdot 8 = x(x+8)$

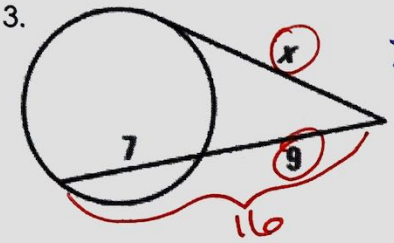
$48 = x^2 + 8x$

$0 = x^2 + 8x - 48$

$0 = (x+12)(x-4)$

$x = -12, 4$

3.

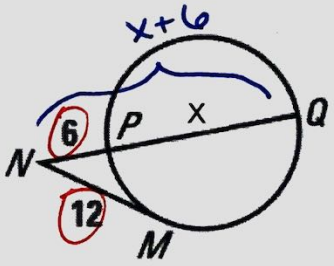


$x^2 = 9 \cdot 16$

$x^2 = 144$

$\boxed{x = 12}$

4.



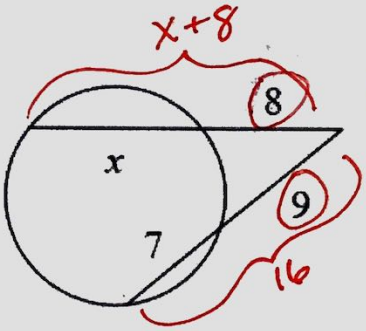
$12^2 = 6(x+6)$

$144 = 6x + 36$

$108 = 6x$

$\boxed{x = 18}$

5.



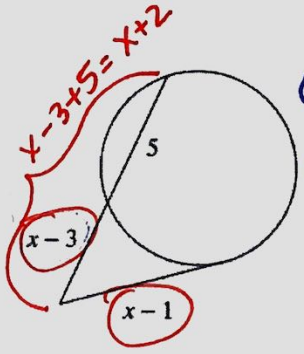
$8(x+8) = 9 \cdot 16$

$8x + 64 = 144$

$8x = 80$

$\boxed{x = 10}$

6.



$(x-1)^2 = (x-3)(x+2)$

$(x-1)(x-1) = (x-3)(x+2)$

$x^2 - 1x - 1x + 1 = x^2 + 2x - 3x - 6$

$x^2 - 2x + 1 = x^2 - 1x - 6$

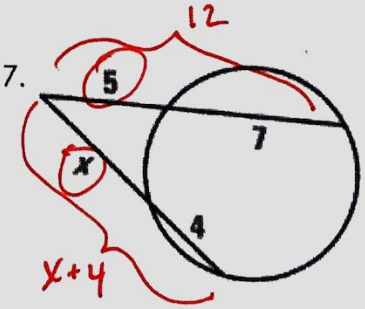
$+2x \qquad +2x$

$1 = x - 6$

$+6 \qquad +6$

$\boxed{7 = x}$

7.



$5 \cdot 12 = x(x+4)$

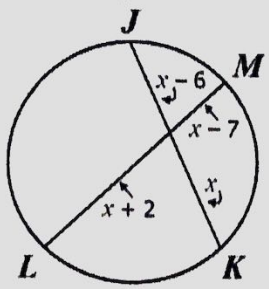
$60 = x^2 + 4x$

$0 = x^2 + 4x - 60$

$0 = (x-6)(x+10)$

$\boxed{x = 6}$ $x = -10$

8.



$(x-7)(x+2) = x(x-6)$

$x^2 + 2x - 7x - 14 = x^2 - 6x$

$-5x - 14 = -6x$

$+5x \qquad +5x$

$-14 = -1x$

$\boxed{x = 14}$