

Review Quiz Sum + Difference

$$\begin{aligned} 1) \quad \sin 75^\circ &= \sin(30^\circ + 45^\circ) \\ &= \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ \\ &= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}} \end{aligned}$$

$$\begin{aligned} 2) \quad \cos \frac{17\pi}{12} &= \cos\left(\frac{15\pi}{12} + \frac{2\pi}{12}\right) = \cos\left(\frac{5\pi}{4} + \frac{\pi}{6}\right) \\ &= \cos \frac{5\pi}{4} \cos \frac{\pi}{6} - \sin \frac{5\pi}{4} \sin \frac{\pi}{6} \\ &= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) \\ &= -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{-\sqrt{6} + \sqrt{2}}{4}} \end{aligned}$$

$$\begin{aligned} 3) \quad \sin \frac{63\pi}{12} &= \sin\left(\frac{9\pi}{12} + \frac{4\pi}{12}\right) = \sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right) \\ &= \sin \frac{3\pi}{4} \cos \frac{\pi}{3} + \cos \frac{3\pi}{4} \sin \frac{\pi}{3} \\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}} \end{aligned}$$

$$\begin{aligned} 4) \quad \cos(-75^\circ) &= \cos(60^\circ - 135^\circ) \\ &= \cos 60^\circ \cos 135^\circ + \sin 60^\circ \sin 135^\circ \\ &= \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{-\sqrt{2} + \sqrt{6}}{4}} \end{aligned}$$

$$\begin{aligned} 5) \quad \cos \frac{5\pi}{12} &= \cos\left(\frac{9\pi}{12} - \frac{4\pi}{12}\right) = \cos\left(\frac{3\pi}{4} - \frac{\pi}{3}\right) \\ &= \cos \frac{3\pi}{4} \cos \frac{\pi}{3} + \sin \frac{3\pi}{4} \sin \frac{\pi}{3} \\ &= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \\ &= -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{-\sqrt{2} + \sqrt{6}}{4}} \end{aligned}$$

$$\begin{aligned}
 6) \quad \tan\left(-\frac{7\pi}{12}\right) &= \tan\left(\frac{2\pi}{12} - \frac{9\pi}{12}\right) = \tan\left(\frac{\pi}{6} - \frac{3\pi}{4}\right) \\
 &= \frac{\tan\frac{\pi}{6} - \tan\frac{3\pi}{4}}{1 + \tan\frac{\pi}{6}\tan\frac{3\pi}{4}} = \frac{\frac{\sqrt{3}}{3} - (-1)}{1 + \left(\frac{\sqrt{3}}{3}\right)(-1)} \\
 &= \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3}} = \frac{\frac{\sqrt{3}}{3} + \frac{3}{3}}{\frac{3 - \sqrt{3}}{3}} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}} \cdot \frac{3}{3} \\
 &= \frac{(3 + \sqrt{3}) \cdot (3 + \sqrt{3})}{(3 - \sqrt{3}) \cdot (3 + \sqrt{3})} = \frac{9 + 3\sqrt{3} + 3\sqrt{3} + 3}{9 - 3} = \frac{12 + 6\sqrt{3}}{6} = \boxed{2 + \sqrt{3}}
 \end{aligned}$$

$$\begin{aligned}
 7) \quad \cos 195^\circ &= \cos(135^\circ + 60^\circ) \\
 &= \cos 135^\circ \cos 60^\circ - \sin 135^\circ \sin 60^\circ \\
 &= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \\
 &= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{-\sqrt{2} - \sqrt{6}}{4}}
 \end{aligned}$$

$$\begin{aligned}
 8) \quad \cos\left(-\frac{\pi}{12}\right) &= \cos\left(\frac{3\pi}{12} - \frac{4\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right) \\
 &= \cos\frac{\pi}{4} \cos\frac{\pi}{3} + \sin\frac{\pi}{4} \sin\frac{\pi}{3} \\
 &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \\
 &= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}
 \end{aligned}$$

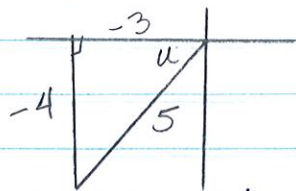
$$\begin{aligned}
 9) \quad \sin(-15^\circ) &= \sin(45^\circ - 60^\circ) \\
 &= \sin 45^\circ \cos 60^\circ - \cos 45^\circ \sin 60^\circ \\
 &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \\
 &= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}
 \end{aligned}$$

$$\begin{aligned}
 10) \quad \sin \frac{5\pi}{12} &= \sin \left(\frac{3\pi}{12} + \frac{2\pi}{12} \right) = \sin \left(\frac{\pi}{4} + \frac{\pi}{6} \right) \\
 &= \sin \frac{\pi}{4} \cos \frac{\pi}{6} + \cos \frac{\pi}{4} \sin \frac{\pi}{6} \\
 &= \left(\frac{\sqrt{2}}{2} \right) \left(\frac{\sqrt{3}}{2} \right) + \left(\frac{\sqrt{2}}{2} \right) \left(\frac{1}{2} \right) \\
 &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}
 \end{aligned}$$

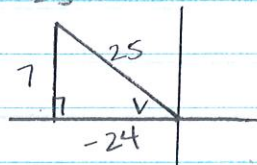
$$\begin{aligned}
 11) \quad \tan \left(-\frac{5\pi}{12} \right) &= \tan \left(\frac{3\pi}{12} - \frac{8\pi}{12} \right) = \tan \left(\frac{\pi}{4} - \frac{2\pi}{3} \right) \\
 &= \frac{\tan \frac{\pi}{4} - \tan \frac{2\pi}{3}}{1 + \tan \frac{\pi}{4} \tan \frac{2\pi}{3}} = \frac{1 - (-\sqrt{3})}{1 + 1 \cdot (-\sqrt{3})} = \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \\
 &= \frac{(1 + \sqrt{3})}{(1 - \sqrt{3})} \cdot \frac{(1 + \sqrt{3})}{(1 + \sqrt{3})} = \frac{1 + \sqrt{3} + \sqrt{3} + 3}{1 + \sqrt{3} - \sqrt{3} - 3} = \frac{4 + 2\sqrt{3}}{-2} = \boxed{-2 - \sqrt{3}}
 \end{aligned}$$

$$\begin{aligned}
 12) \quad \cos \frac{7\pi}{12} &= \cos \left(\frac{3\pi}{12} + \frac{4\pi}{12} \right) = \cos \left(\frac{\pi}{4} + \frac{\pi}{3} \right) \\
 &= \cos \frac{\pi}{4} \cos \frac{\pi}{3} - \sin \frac{\pi}{4} \sin \frac{\pi}{3} \\
 &= \left(\frac{\sqrt{2}}{2} \right) \left(\frac{1}{2} \right) - \left(\frac{\sqrt{2}}{2} \right) \left(\frac{\sqrt{3}}{2} \right) \\
 &= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}
 \end{aligned}$$

$$(3) \tan u = \frac{4}{3} \quad \pi < u < \frac{3\pi}{2}$$



$$\sin v = \frac{7}{25} \quad \frac{\pi}{2} < v < \pi$$



$$\begin{aligned} a) \sin(u-v) &= \sin u \cos v - \cos u \sin v \\ &= \left(-\frac{4}{5}\right)\left(-\frac{24}{25}\right) - \left(-\frac{3}{5}\right)\left(\frac{7}{25}\right) \\ &= \frac{96}{125} + \frac{21}{125} = \boxed{\frac{117}{125}} \end{aligned}$$

$$\begin{aligned} b) \cos(u-v) &= \cos u \cos v + \sin u \sin v \\ &= \left(-\frac{3}{5}\right)\left(-\frac{24}{25}\right) + \left(-\frac{4}{5}\right)\left(\frac{7}{25}\right) \\ &= \frac{72}{125} - \frac{28}{125} = \boxed{\frac{44}{125}} \end{aligned}$$

$$\begin{aligned} c) \tan(u+v) &= \frac{\tan u + \tan v}{1 - \tan u \tan v} \\ &= \frac{\frac{4}{3} + \frac{-7}{24}}{1 - \frac{4}{3}\left(\frac{-7}{24}\right)} = \frac{\frac{32}{24} - \frac{7}{24}}{1 + \frac{28}{72}} = \frac{\frac{25}{24}}{\frac{72}{72} + \frac{28}{72}} \\ &= \frac{\frac{25}{24}}{\frac{100}{72}} = \frac{25}{24} \cdot \frac{72}{100} = \boxed{\frac{3}{4}} \end{aligned}$$