

Sum and Difference Identities WS 3 – Tangent

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

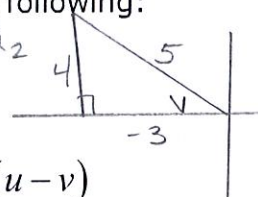
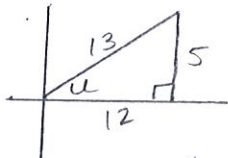
Find the exact value of each expression.

$$\begin{aligned} 1. \quad \tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right) &= \frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{3}}{1 - \tan\frac{\pi}{4} \tan\frac{\pi}{3}} \\ &= \frac{1 + \sqrt{3}}{1 - (1)(\sqrt{3})} = \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \cdot \frac{(1 + \sqrt{3})}{(1 + \sqrt{3})} = \frac{1 + 2\sqrt{3} + 3}{1 - 3} \\ &= \frac{4 + 2\sqrt{3}}{-2} = \boxed{-2 - \sqrt{3}} \end{aligned}$$

$$\begin{aligned} 2. \quad \tan\frac{\pi}{4} + \tan\frac{\pi}{3} \\ &= \boxed{1 + \sqrt{3}} \end{aligned}$$

Find the exact value of the trigonometric function given the following:

$$\sin u = \frac{5}{13}, \quad \boxed{0 < u < \frac{\pi}{2}} \quad Q_1 \quad \text{and} \quad \cos v = -\frac{3}{5}, \quad \boxed{\frac{\pi}{2} < v < \pi} \quad Q_2$$



3. $\tan(u + v)$

$$\begin{aligned} \frac{\tan u + \tan v}{1 - \tan u \tan v} &= \frac{\left(\frac{5}{12}\right) + \left(-\frac{4}{3}\right)}{1 - \left(\frac{5}{12}\right)\left(-\frac{4}{3}\right)} \\ &= \frac{\frac{5}{12} - \frac{16}{12}}{\frac{36}{36} + \frac{20}{36}} = \frac{-\frac{11}{12}}{\frac{56}{36}} = \frac{-11}{12} \cdot \frac{36}{56} = \boxed{\frac{-33}{56}} \end{aligned}$$

4. $\tan(u - v)$

$$\begin{aligned} \frac{\tan u - \tan v}{1 + \tan u \tan v} &= \frac{\frac{5}{12} - \left(-\frac{4}{3}\right)}{1 + \left(\frac{5}{12}\right)\left(-\frac{4}{3}\right)} \\ &= \frac{\frac{5}{12} + \frac{16}{12}}{\frac{36}{36} - \frac{20}{36}} = \frac{\frac{21}{12}}{\frac{16}{36}} = \frac{21}{12} \cdot \frac{36}{16} = \boxed{\frac{63}{16}} \end{aligned}$$

Use the sum and difference formulas to write the expression as the sine, cosine, or tangent of a single angle.

5. $\cos 40^\circ \cos 15^\circ - \sin 40^\circ \sin 15^\circ = \cos(40^\circ + 15^\circ) = \boxed{\cos(55^\circ)}$

6. $\sin 340^\circ \cos 50^\circ - \cos 340^\circ \sin 50^\circ = \sin(340^\circ - 50^\circ) = \boxed{\sin 290^\circ}$

7. $\frac{\tan 325^\circ - \tan 86^\circ}{1 + \tan 325^\circ \tan 86^\circ} = \tan(325^\circ - 86^\circ) = \boxed{\tan 239^\circ}$