PreCalculus

Name Key

Sum and Difference Identities WS 2 – Cosine

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

Find the exact value of each expression.

1. $\cos\left(\frac{\pi}{6}+\frac{\pi}{3}\right)$ ⟷ compare ⟷ 2. $\cos\frac{\pi}{6}+\cos\frac{\pi}{3} = \frac{\sqrt{3}}{2}+\frac{1}{2}$

$= \cos\frac{\pi}{6}\cos\frac{\pi}{3} - \sin\frac{\pi}{6}\sin\frac{\pi}{3}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = \boxed{0}$

$= \boxed{\frac{\sqrt{3}+1}{2}}$

Use the sum and difference formulas to find the exact values of the cosine of the angle.

4. $75^\circ = 30^\circ + 45^\circ$

$\cos 75^\circ = \cos(30^\circ + 45^\circ)$

$= \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \sqrt{6}/4 - \sqrt{2}/4 = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

5. $105^\circ = 60^\circ + 45^\circ$

$\cos 105^\circ = \cos(60^\circ + 45^\circ)$

$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

6. $195^\circ = 225^\circ - 30^\circ$

$\cos 195^\circ = \cos(225^\circ - 30^\circ)$

$= \cos 225 \cos 30 + \sin 225 \sin 30$

$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$

7. $\frac{11\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$

$\cos\frac{11\pi}{12} = \cos\left(\frac{3\pi}{4}+\frac{\pi}{6}\right)$

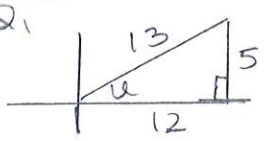
$= \cos\frac{3\pi}{4}\cos\frac{\pi}{6} - \sin\frac{3\pi}{4}\sin\frac{\pi}{6}$

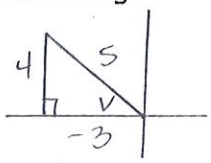
$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$

Find the exact value of the trigonometric function given the following:

$\sin u \overset{\text{opp}}{\underset{\text{hyp}}{=}} \frac{5}{13}$, $0 < u < \frac{\pi}{2}$ (Q_1) and $\cos v \overset{\text{adj}}{\underset{\text{hyp}}{=}} \frac{3}{5}$, $\frac{\pi}{2} < v < \pi$ (Q_2)





8. $\cos(v-u)$

$= \cos v \cos u + \sin v \sin u$

$= \left(-\frac{3}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{5}{13}\right)$

$= -\frac{36}{65} + \frac{20}{65} = \boxed{\frac{-16}{65}}$

9. $\cos(u+v)$

$= \cos u \cos v - \sin u \sin v$

$= \left(\frac{12}{13}\right)\left(-\frac{3}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{4}{5}\right)$

$= \frac{-36}{65} - \frac{20}{65} = \boxed{\frac{-56}{65}}$