Review - Sum + Difference Identities

1) $\sin 97^\circ \cos 43^\circ + \cos 97^\circ \sin 43^\circ = \sin(97^\circ + 43^\circ) = \boxed{\sin(140^\circ)}$

2) $\cos 72^\circ \cos 130^\circ + \sin 72^\circ \sin 130^\circ = \cos(72^\circ - 130^\circ) = \boxed{\cos(58^\circ)}$

3) $\dfrac{\tan 140^\circ - \tan 60^\circ}{1 + \tan 40^\circ \tan 60^\circ} = \tan(140^\circ - 60^\circ) = \boxed{\tan 80^\circ}$

4) $\sin\frac{\pi}{5}\cos\frac{2\pi}{3} - \cos\frac{\pi}{5}\sin\frac{2\pi}{3} = \sin\left(\frac{\pi}{5} - \frac{2\pi}{3}\right)$

$= \sin\left(\frac{3\pi}{15} - \frac{10\pi}{15}\right) = \sin\left(-\frac{7\pi}{15}\right) = \boxed{-\sin\frac{7\pi}{15}}$

5) $\cos\frac{\pi}{6}\cos\frac{\pi}{7} - \sin\frac{\pi}{6}\sin\frac{\pi}{7} = \cos\left(\frac{\pi}{6} + \frac{\pi}{7}\right)$

$= \cos\left(\frac{7\pi}{42} + \frac{6\pi}{42}\right) = \boxed{\cos\frac{13\pi}{42}}$

6) $\dfrac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{4}} = \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \tan\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right)$

$= \boxed{\tan\frac{7\pi}{12}}$

7) $\tan(-105^\circ) = \tan(45^\circ - 150^\circ) = \dfrac{\tan 45^\circ - \tan 150^\circ}{1 + \tan 45^\circ \tan 150^\circ}$

$= \dfrac{1 - -\frac{\sqrt{3}}{3}}{1 + (1)\left(-\frac{\sqrt{3}}{3}\right)} = \dfrac{\left(1 + \frac{\sqrt{3}}{3}\right)3}{\left(1 - \frac{\sqrt{3}}{3}\right)3} = \dfrac{(3+\sqrt{3})}{(3-\sqrt{3})} \cdot \dfrac{(3+\sqrt{3})}{(3+\sqrt{3})} = \dfrac{9 + 6\sqrt{3} + 3}{9 - 3}$

$= \dfrac{12 + 6\sqrt{3}}{6} = \dfrac{12}{6} + \dfrac{6\sqrt{3}}{6} = \boxed{2 + \sqrt{3}}$

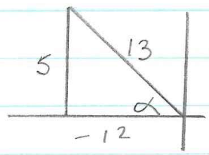
8) $\sin 345^\circ = \sin(300^\circ + 45^\circ)$

$= \sin 300^\circ \cos 45^\circ + \cos 300^\circ \sin 45^\circ$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \dfrac{-\sqrt{6}}{4} + \dfrac{\sqrt{2}}{4} = \boxed{\dfrac{-\sqrt{6} + \sqrt{2}}{4}}$

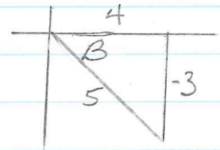
#9-11



$\csc\alpha = \frac{13}{5}$

$\sin\alpha = \frac{5}{13}$

$\cos\alpha = \frac{-12}{13}$



$\tan\beta = -\frac{3}{4}$

$\sin\beta = -\frac{3}{5}$

$\cos\beta = \frac{4}{5}$

9) $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \left(\frac{5}{13}\right)\left(\frac{4}{5}\right) - \left(-\frac{12}{13}\right)\left(-\frac{3}{5}\right)$

$= \frac{20}{65} - \frac{36}{65} = \boxed{\frac{-16}{65}}$

10) $\cos(\beta+\alpha) = \cos\beta\cos\alpha - \sin\beta\sin\alpha$

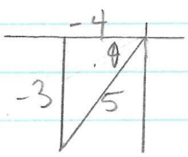
$= \left(\frac{4}{5}\right)\left(-\frac{12}{13}\right) - \left(-\frac{3}{5}\right)\left(\frac{5}{13}\right)$

$= \frac{-48}{65} + \frac{15}{65} = \boxed{\frac{-33}{65}}$

11) $\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \frac{-\frac{5}{12} - \left(-\frac{3}{4}\right)}{1+\left(-\frac{5}{12}\right)\left(-\frac{3}{4}\right)}$

$= \frac{-\frac{5}{12}+\frac{3}{4}}{1+\frac{15}{48}} = \frac{-\frac{5}{12}+\frac{9}{12}}{\frac{48}{48}+\frac{15}{48}} = \frac{\frac{4}{12}}{\frac{63}{48}} = \frac{4}{\cancel{12}_{1}} \cdot \frac{\cancel{48}^{4}}{63} = \boxed{\frac{16}{63}}$

#12-13



$\sin\theta = -\frac{3}{5}$ $\tan\theta = \frac{3}{4}$

$\cos\theta = -\frac{4}{5}$

12) $\cos\left(\theta+\frac{\pi}{3}\right) = \cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3}$

$= \left(-\frac{4}{5}\right)\left(\frac{1}{2}\right) - \left(-\frac{3}{5}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= -\frac{4}{10} + \frac{3\sqrt{5}}{10} = \boxed{\frac{-4+3\sqrt{5}}{10}}$

13) $\tan 2\theta = \tan(\theta+\theta) = \frac{\tan\theta+\tan\theta}{1-\tan\theta\tan\theta} = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{6}{4}}{1-\frac{9}{16}} = \frac{\frac{6}{4}}{\frac{16}{16}-\frac{9}{16}} = \frac{\frac{6}{4}}{\frac{7}{16}} = \frac{6}{\cancel{4}} \cdot \frac{\cancel{16}^{4}}{7} = \boxed{\frac{24}{7}}$

14) $\sin(\pi - x) = \sin x$

$\sin(\pi - x) = \sin\pi\cos x - \cos\pi\sin x$

$= (0)\cancel{(\cos x)} - (-1)(\sin x) = \boxed{\sin x}$ ✓

15) $\sin\left(\frac{3\pi}{2} + x\right) = -\cos x$

$\sin\frac{3\pi}{2}\cos x + \cos\frac{3\pi}{2}\sin x$

$(-1)\cos x + (0)\sin x = \boxed{-\cos x}$ ✓

16) $\cos(30° - x) + \cos(30° + x) = \sqrt{3}\cos x$

$= \cos 30°\cos x + \cancel{\sin 30°\sin x} + \cos 30°\cos x - \cancel{\sin 30°\sin x}$

$= 2\cos 30°\cos x$

$= 2\left(\frac{\sqrt{3}}{2}\right)\cos x$

$= \boxed{\sqrt{3}\cos x}$ ✓

17) $\frac{\sin(\beta - \alpha)}{\sin\alpha\sin\beta} = \cot\alpha - \cot\beta$

$= \frac{\sin\beta\cos\alpha - \cos\beta\sin\alpha}{\sin\alpha\sin\beta}$

$= \frac{\cancel{\sin\beta}\cos\alpha}{\sin\alpha\cancel{\sin\beta}} - \frac{\cos\beta\cancel{\sin\alpha}}{\cancel{\sin\alpha}\sin\beta}$

$= \frac{\cos\alpha}{\sin\alpha} - \frac{\cos\beta}{\sin\beta}$

$= \boxed{\cot\alpha - \cot\beta}$ ✓

18) $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos\alpha\cos\beta$

$= \cos\alpha\cos\beta - \cancel{\sin\alpha\sin\beta} + \cos\alpha\cos\beta + \cancel{\sin\alpha\sin\beta}$

$= \boxed{2\cos\alpha\cos\beta}$ ✓

19) $\sin(x+\frac{\pi}{6}) - \sin(x-\frac{\pi}{6}) = \frac{1}{2}$

$\sin x\cos\frac{\pi}{6} + \cos x\sin\frac{\pi}{6} - \left(\sin x\cos\frac{\pi}{6} - \cos x\sin\frac{\pi}{6}\right) = \frac{1}{2}$

$\cancel{\sin x\cos\frac{\pi}{6}} + \cos x\sin\frac{\pi}{6} - \cancel{\sin x\cos\frac{\pi}{6}} + \cos x\sin\frac{\pi}{6} = \frac{1}{2}$

$2\cos x\sin\frac{\pi}{6} = \frac{1}{2}$

$2\cos x(\frac{1}{2}) = \frac{1}{2}$

$\cos x = \frac{1}{2}$ $\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

20) $\tan(x+\pi) + 2\sin(x+\pi) = 0$

$\frac{\tan x + \tan\pi}{1-\tan x\tan\pi} + 2(\sin x\cos\pi + \cos x\sin\pi) = 0$

$\frac{\tan x + 0}{1-\cancel{\tan x}(0)} + 2\left(\sin x(-1) + \cancel{\cos x(0)}\right) = 0$

$\tan x - 2\sin x = 0$

$\cos x\left(\frac{\sin x}{\cos x} - 2\sin x = 0\right)$

$\sin x - 2\sin x\cos x = 0$

$\sin x(1-2\cos x) = 0$

$\sin x = 0$ $1-2\cos x = 0$

$\boxed{x = 0\pi, \pi}$ $-2\cos x = -1$

$\cos x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

21) $\sin(x+\frac{\pi}{2}) - \cos(x+\frac{3\pi}{2}) = 0$

$\sin x\cos\frac{\pi}{2} + \cos x\sin\frac{\pi}{2} - \left(\cos x\cos\frac{3\pi}{2} - \sin x\sin\frac{3\pi}{2}\right) = 0$

$\sin x\cos\frac{\pi}{2} + \cos x\sin\frac{\pi}{2} - \cos x\cos\frac{3\pi}{2} + \sin x\sin\frac{3\pi}{2} = 0$

$\cancel{\sin x(0)} + \cos x(1) - \cancel{\cos x(0)} + \sin x(-1) = 0$

$\cos x - \sin x = 0$

$\cos x = \sin x$

Where does $\cos x = \sin x$? $\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$