Solving Trig Equations with Pythagorean Substitutions

$\sin^2 x + \cos^2 x = 1$
$1 + \tan^2 x = \sec^2 x$
$1 + \cot^2 x = \csc^2 x$

$\sin^2 x + \cos^2 x = 1$
$\cos^2 x = 1 - \sin^2 x$

Solve over $[0, 2\pi)$.

- no gcf
- can't factor the way it is
- there is more than 1 trig function

- $\cos^2 x + \sin x = 1$ ← Replace $\cos^2 x$ with $1 - \sin^2 x$

$1 - \sin^2 x + \sin x = 1$ ← cancel out the 1 on both sides

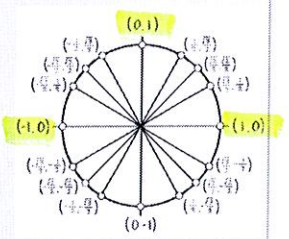
$-\sin^2 x + \sin x = 0$ ← factor out $-\sin x$

$-\sin x(\sin x - 1) = 0$

$-\sin x = 0 \quad \sin x - 1 = 0$

$\sin x = 0 \quad \sin x = 1$

$x = 0, \pi$ $\quad$ $x = \frac{\pi}{2}$



* No extraneous solutions with sin + cos.

$\sin^2 x + \cos^2 x = 1$
$\sin^2 x = 1 - \cos^2 x$

Solve over $[0, 2\pi)$.

* substitute for $\sin^2 x$ or $\cos^2 x$

- $\sin^2 x = \cos^2 x$

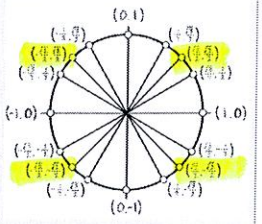
$1 - \cos^2 x = \cos^2 x$
$+\cos^2 x \quad +\cos^2 x$

$\frac{1}{2} = \frac{2\cos^2 x}{2}$

$\frac{1}{2} = \cos^2 x$

$\pm\frac{1}{\sqrt{2}} = \cos x$

$\cos x = \pm\frac{\sqrt{2}}{2}$



$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Solving Trig Equations with Pythagorean Substitutions

Solve over $[0, 2\pi)$.

$\sin^2x + \cos^2x = 1$

$\sin^2x = 1 - \cos^2x$

- $2\sin^2x = 2 + \cos x$ Pyth. subst. for $\sin^2x$

$2(1 - \cos^2x) = 2 + \cos x$ ← distribute 2

$2 - 2\cos^2x = 2 + \cos x$ ← set equation to zero

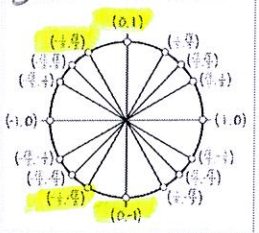
$-2\cos^2x - \cos x = 0$ ← factor out $\cos x$

$\cos x(2\cos x + 1) = 0$

$\cos x = 0$ $\quad 2\cos x + 1 = 0$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$



$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$

* sin + cos are never undefined so no extraneous solutions.

Solve over $[0, 2\pi)$.

Mismatched trig functions so replace $\csc^2x$

$\csc^2x = \cot^2x + 1$

- $\csc^2x = 2\cot x$

$\cot^2x + 1 = 2\cot x$

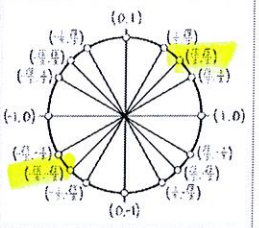
$\cot^2x - 2\cot x + 1 = 0$

$(\cot x - 1)(\cot x - 1) = 0$

$\cot x - 1 = 0$

$\cot x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$



* not extraneous because $\frac{\pi}{4}$ & $\frac{5\pi}{4}$ are not quadrantal.