

Multiple Angle Practice

21. $\sin^2 x = 3 \cos^2 x$

$$1 - \cos^2 x = 3 \cos^2 x$$

$$1 = 4 \cos^2 x$$

$$\sqrt{\frac{1}{4}} = \sqrt{\cos^2 x}$$

$$\pm \frac{1}{2} = \cos x$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

22. $\tan 3x (\tan x - 1) = 0$

$$\tan 3x = 0$$

$$\tan x - 1 = 0$$

$\frac{1}{3}$

$$3x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$$

$\underbrace{\hspace{1.5cm}}_{+2\pi} \quad \underbrace{\hspace{1.5cm}}_{+2\pi}$

$$\tan x = 1$$

$$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

π

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

23. $(3 \tan^2 x - 1)(\tan^2 x - 3) = 0$

$$3 \tan^2 x = 1$$

$$\tan^2 x - 3 = 0$$

$$\tan^2 x = \frac{1}{3}$$

$$\tan^2 x = 3$$

$$\tan x = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$$

$$\tan x = \pm \sqrt{3}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

24. $\cos 2x (2 \cos x + 1) = 0$

$$\cos 2x = 0$$

$$2 \cos x + 1 = 0$$

$\frac{1}{2}$

$$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$\underbrace{\hspace{1.5cm}}_{+4\pi} \quad \underbrace{\hspace{1.5cm}}_{+4\pi}$

$$2 \cos x = -1$$

$$\cos x = -\frac{1}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$25. \quad \cos^3 x = \cos x$$

$$\cos^3 x - \cos x = 0$$

$$\cos x (\cos^2 x - 1) = 0$$

$$\cos x = 0 \quad \cos^2 x - 1 = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sqrt{\cos^2 x} = \sqrt{1}$$

$$\cos x = \pm 1$$

$$x = 0, \pi$$

$$27. \quad 3 \tan^3 x = \tan x$$

$$3 \tan^3 x - \tan x = 0$$

$$\tan x (3 \tan^2 x - 1) = 0$$

$$\tan x = 0$$

$$3 \tan^2 x - 1 = 0$$

$$x = 0, \pi$$

$$3 \tan^2 x = 1$$

$$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$$

$$\tan x = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Replace \downarrow

$$28. \quad 2 \sin^2 x = 2 + \cos x$$

$$2(1 - \cos^2 x) = 2 + \cos x$$

$$2 - 2 \cos^2 x = 2 + \cos x$$

$$2 - 2 \cos^2 x - 2 - \cos x = 0$$

$$-2 \cos^2 x - \cos x = 0$$

$$-\cos x (2 \cos x + 1) = 0$$

$$-\cos x = 0 \quad 2 \cos x + 1 = 0$$

$$\cos x = 0$$

$$2 \cos x = -1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos x = -\frac{1}{2}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$31. \quad 2 \sin x + \csc x = 0$$

$$2 \sin x + \frac{1}{\sin x} = 0$$

$$\sin x \left(2 \sin x = -\frac{1}{\sin x} \right)$$

$$2 \sin^2 x = -1$$

$$\sqrt{\sin^2 x} = \sqrt{-\frac{1}{2}} \quad \leftarrow \text{imaginary}$$

$$\sin x = \boxed{\text{no solution}}$$

$$32. \quad \sin 2x = -\frac{\sqrt{3}}{2}$$

$$\frac{1}{2} \left[2x = \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3} \right]$$

$+ 6\pi/3$

$$x = \frac{4\pi}{6}, \frac{5\pi}{6}, \frac{10\pi}{6}, \frac{11\pi}{6}$$

$\frac{2\pi}{3} \quad \frac{5\pi}{3}$

$$33. \quad \csc x + \cot x = 1$$

$$\frac{1}{\sin x} + \frac{\cos x}{\sin x} = 1$$

$$\frac{1 + \cos x}{\sin x} = 1$$

$$(1 + \cos x)^2 = (\sin x)^2$$

$$(1 + \cos x)(1 + \cos x) = \sin^2 x$$

$$1 + 2\cos x + \cos^2 x = 1 - \cos^2 x$$

$$2\cos^2 x + 2\cos x = 0$$

$$2\cos x(\cos x + 1) = 0$$

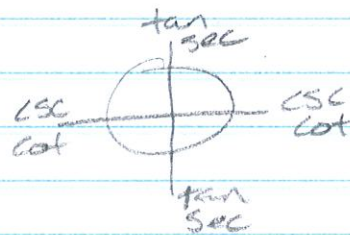
$$2\cos x = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos x = -1$$

$$x = \pi$$



34. $\tan 3x = 1$

$$\frac{1}{3} \left[3x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}, \frac{21\pi}{4} \right]$$

$\underbrace{\hspace{1.5cm}}_{+8\pi/4} \quad \underbrace{\hspace{1.5cm}}_{+8\pi/4}$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{21\pi}{12}$$

$\frac{3\pi}{4} \qquad \frac{7\pi}{4}$

36. $\sec 4x = 2$

$\cos 4x = \frac{1}{2}$

$$\frac{1}{4} \left[4x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{2\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}, \frac{19\pi}{3}, \frac{23\pi}{3} \right]$$

$\underbrace{\hspace{1.5cm}}_{+6\pi/3} \quad \underbrace{\hspace{1.5cm}}_{+6\pi/3} \quad \underbrace{\hspace{1.5cm}}_{+6\pi/3}$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{2\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$$

37. $\frac{1 + \cos x}{1 - \cos x} = 0$ multiply $(1 - \cos x)$ to both sides

$1 + \cos x = 0$

$\cos x = -1 \quad \boxed{x = \pi}$

38. $2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$2\sin x = -1 \quad \sin x = -1$

$\sin x = -\frac{1}{2}$

$\boxed{x = \frac{3\pi}{2}}$

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

replace

39.

$$2 \sec^2 x + \tan^2 x - 3 = 0$$

$$2(1 + \tan^2 x) + \tan^2 x - 3 = 0$$

$$2 + 2 \tan^2 x + \tan^2 x - 3 = 0$$

$$3 \tan^2 x - 1 = 0$$

$$3 \tan^2 x = 1$$

$$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$$

$$\tan x = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

40. $\cos x + \sin x \tan x = 2$

$$\frac{(\cos x)}{(\cos x)} \cos x + \frac{\sin^2 x}{\cos x} = 2$$

$$\frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} = 2$$

$$\frac{\cos^2 x + \sin^2 x}{\cos x} = 2$$

$$\frac{1}{\cos x} = \frac{2}{1} \quad \text{cross multiply}$$

$$2 \cos x = 1$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$