5-2 Verifying Trig Identities

1. $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

$\tan^2\theta \cos^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta$

$\boxed{\sin^2\theta}$ ✓

2. $\sec^2\theta(1 - \cos^2\theta) = \tan^2\theta$

$\sec^2\theta - \sec^2\theta\cos^2\theta$

$\sec^2\theta - \frac{1}{\cos^2\theta} \cdot \cos^2\theta$

$\sec^2\theta - 1$

$\boxed{\tan^2\theta}$ ✓

3. $\sin\theta - \sin\theta\cos^2\theta = \sin^3\theta$

$\sin\theta(1 - \cos^2\theta)$

$\sin\theta(\sin^2\theta)$

$\boxed{\sin^3\theta}$ ✓

4. $\csc\theta - \cos\theta\cot\theta = \sin\theta$

$\frac{1}{\sin\theta} - \cos\theta \cdot \frac{\cos\theta}{\sin\theta}$

$\frac{1 - \cos^2\theta}{\sin\theta}$

$\frac{\sin^2\theta}{\sin\theta}$

$\boxed{\sin\theta}$ ✓

5. $\cot^2\theta\csc^2\theta - \cot^2\theta = \cot^4\theta$

$\cot^2\theta(\csc^2\theta - 1)$

$\cot^2\theta\cot^2\theta$

$\boxed{\cot^4\theta}$ ✓

6. $\tan\theta \csc^2\theta - \tan\theta = \cot\theta$

$\tan\theta(\csc^2\theta - 1)$

$\tan\theta \cot^2\theta$

$\frac{1}{\cot\theta} \cdot \cot^2\theta$

$\boxed{\cot\theta}$ ✓

7. $\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta$

multi. by cos: $(\cos\theta)$ / $(\cos\theta)$; mult. by sin: $(\sin\theta)$ / $(\sin\theta)$

$= \frac{\sec\theta\cos\theta - \sin^2\theta}{\sin\theta\cos\theta}$

$= \frac{\frac{1}{\cos\theta}\cdot\cos\theta - \sin^2\theta}{\sin\theta\cos\theta}$

$= \frac{1 - \sin^2\theta}{\sin\theta\cos\theta}$

$= \frac{\cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$= \boxed{\cot\theta}$ ✓

8. $\frac{\sin\theta}{1-\cos\theta}\frac{(1+\cos\theta)}{} + \frac{1-\cos\theta}{\sin\theta} = 2\csc\theta$

mult. by $1+\cos\theta$

$\frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta} + \frac{1-\cos\theta}{\sin\theta}$

$\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta} + \frac{1-\cos\theta}{\sin\theta}$

$\frac{1+\cos\theta}{\sin\theta} + \frac{1-\cos\theta}{\sin\theta}$

$\frac{1+\cos\theta+1-\cos\theta}{\sin\theta}$

$\frac{2}{\sin\theta} = \boxed{2\csc\theta}$ ✓

9. $\dfrac{\cos\theta}{1+\sin\theta} + \tan\theta = \sec\theta$

$= \dfrac{(\cos\theta)}{(\cos\theta)}\dfrac{\cos\theta}{1+\sin\theta} + \dfrac{\sin\theta}{\cos\theta}\dfrac{(1+\sin\theta)}{(1+\sin\theta)}$

$= \dfrac{\cos^2\theta}{\cos\theta(1+\sin\theta)} + \dfrac{\sin\theta(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$

$= \dfrac{\cos^2\theta + \sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)}$

$= \dfrac{1+\sin\theta}{\cos\theta(1+\sin\theta)}$

$= \dfrac{1}{\cos\theta} = \boxed{\sec\theta}$ ✓

10. $\dfrac{\sin\theta}{1-\cot\theta} + \dfrac{\cos\theta}{1-\tan\theta} = \sin\theta + \cos\theta$

$= \dfrac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}} + \dfrac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}$ Rewrite using common denom.

$= \dfrac{\sin\theta}{\frac{\sin\theta}{\sin\theta} - \frac{\cos\theta}{\sin\theta}} + \dfrac{\cos\theta}{\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}$

$= \dfrac{\sin\theta}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \dfrac{\cos\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$= \sin\theta \cdot \dfrac{\sin\theta}{\sin\theta-\cos\theta} + \cos\theta \cdot \dfrac{\cos\theta}{\cos\theta-\sin\theta}$

$= \dfrac{\sin^2\theta}{\sin\theta-\cos\theta} - \dfrac{\cos^2\theta}{-\cos\theta+\sin\theta}$

$= \dfrac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta} = \dfrac{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}{\sin\theta-\cos\theta} = \boxed{\sin\theta+\cos\theta}$ ✓

11. $\frac{1}{1-\tan^2\theta} + \frac{1}{1-\cot^2\theta}$

$\frac{1}{1-\frac{\sin^2\theta}{\cos^2\theta}} + \frac{1}{1-\frac{\cos^2\theta}{\sin^2\theta}}$

$\frac{1}{\frac{\cos^2\theta}{\cos^2\theta}-\frac{\sin^2\theta}{\cos^2\theta}} + \frac{1}{\frac{\sin^2\theta}{\sin^2\theta}-\frac{\cos^2\theta}{\sin^2\theta}}$

$\frac{1}{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}} + \frac{1}{\frac{\sin^2\theta-\cos^2\theta}{\sin^2\theta}}$

$1\cdot\frac{\cos^2\theta}{\cos^2\theta-\sin^2\theta} + 1\cdot\frac{\sin^2\theta}{(\sin^2\theta-\cos^2\theta)(-1)}$ (-1)

$\frac{\cos^2\theta}{\cos^2\theta-\sin^2\theta} - \frac{\sin^2\theta}{\cos^2\theta-\sin^2\theta}$

$\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta-\sin^2\theta} = \boxed{1}$ ✓

12. $\frac{1}{\csc\theta+1}\frac{(\csc\theta-1)}{(\csc\theta-1)} + \frac{1}{\csc\theta-1}\frac{(\csc\theta+1)}{(\csc\theta+1)} = 2\sec^2\theta\sin\theta$ multiply by conjugates

$\frac{\csc\theta-1}{\csc^2\theta-1} + \frac{\csc\theta+1}{\csc^2\theta-1}$

$\frac{\csc\theta-1+\csc\theta+1}{\cot^2\theta}$

$\frac{2\csc\theta}{\cot^2\theta}$

$\frac{2/\sin\theta}{\cos^2\theta/\sin^2\theta}$

$\frac{2}{\sin\theta}\cdot\frac{\sin^2\theta}{\cos^2\theta}$

$\frac{2\sin\theta}{\cos^2\theta} = \boxed{2\sec^2\theta\sin\theta}$ ✓

13. $(\csc\theta - \cot\theta)(\csc\theta + \cot\theta) = 1$

$= \csc^2\theta - \cot^2\theta$

$= \cot^2\theta + 1 - \cot^2\theta$

$= \boxed{1}$ ✓

14. $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$

$(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$(\cos^2\theta - \sin^2\theta)(1)$

$\boxed{\cos^2\theta - \sin^2\theta}$ ✓

15. $\frac{1}{1-\sin\theta}\frac{(1+\sin\theta)}{(1+\sin\theta)} + \frac{1}{1+\sin\theta}\frac{(1-\sin\theta)}{(1-\sin\theta)} = 2\sec^2\theta$ mult. by conjugates

$= \frac{1+\sin\theta + 1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{2}{1-\sin^2\theta}$

$= \frac{2}{\cos^2\theta}$

$= \boxed{2\sec^2\theta}$ ✓

16. $\frac{\cos\theta}{1+\sin\theta}\frac{(1-\sin\theta)}{(1-\sin\theta)} + \frac{\cos\theta}{1-\sin\theta}\frac{(1+\sin\theta)}{(1+\sin\theta)} = 2\sec\theta$ mult. by conjugate

$\frac{\cos\theta(1-\sin\theta) + \cos\theta(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$\frac{\cos\theta - \cos\theta\sin\theta + \cos\theta + \cos\theta\sin\theta}{1-\sin^2\theta}$

$\frac{2\cos\theta}{\cos^2\theta}$

$\frac{2}{\cos\theta}$

$\boxed{2\sec\theta}$ ✓

17. $\csc^4\theta - \cot^4\theta = 2\cot^2\theta + 1$

$(\csc^2\theta - \cot^2\theta)(\csc^2\theta + \cot^2\theta)$

$(1)\ (1 + \cot^2\theta + \cot^2\theta)$

$\boxed{1 + 2\cot^2\theta}$ ✓

18. $\dfrac{\csc^2\theta + 2\csc\theta - 3}{\csc^2\theta - 1} = \dfrac{\csc\theta + 3}{\csc\theta + 1}$

$= \dfrac{(\csc\theta + 3)(\csc\theta - 1)}{(\csc\theta + 1)(\csc\theta - 1)}$

$= \boxed{\dfrac{\csc\theta + 3}{\csc\theta + 1}}$ ✓

20. $(\csc\theta + \cot\theta)(1 - \cos\theta) = \sin\theta$

$= \csc\theta - \cos\theta\csc\theta + \cot\theta - \cos\theta\cot\theta$

$= \dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta} - \cos\theta \cdot \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{1 - \cos\theta + \cos\theta - \cos^2\theta}{\sin\theta}$

$= \dfrac{1 - \cos^2\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta}{\sin\theta}$

$= \boxed{\sin\theta}$ ✓

21. $\sin^2\theta \tan^2\theta = \tan^2\theta - \sin^2\theta$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{1}\,\frac{(\cos^2\theta)}{(\cos^2\theta)}$$

$$= \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}$$

$$= \frac{\sin^2\theta \sin^2\theta}{\cos^2\theta}$$

$$= \boxed{\sin^2\theta\tan^2\theta} \quad \checkmark$$

22. $\dfrac{1-\tan^2\theta}{1-\cot^2\theta} = \dfrac{\cos^2\theta - 1}{\cos^2\theta}$

$$= \frac{1 - \frac{\sin^2\theta}{\cos^2\theta}}{1 - \frac{\cos^2\theta}{\sin^2\theta}}$$

mult. by $\cos^2\theta$

mult. by $\sin^2\theta$

$$= \frac{\frac{\cos^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta} \cdot \frac{\sin^2\theta}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta} \cdot \frac{\sin^2\theta}{-(\cos^2\theta - \sin^2\theta)}$$

$$= -\frac{\sin^2\theta}{\cos^2\theta} = \boxed{\frac{\cos^2\theta - 1}{\cos^2\theta}} \quad \checkmark$$

23. $\dfrac{1+\csc\theta}{\sec\theta} = \cos\theta + \cot\theta$

$= \dfrac{1}{\sec\theta} + \dfrac{\csc\theta}{\sec\theta}$

$= \cos\theta + \dfrac{1/\sin\theta}{1/\cos\theta}$

$= \cos\theta + \dfrac{1}{\sin\theta} \cdot \dfrac{\cos\theta}{1}$

$= \cos\theta + \dfrac{\cos\theta}{\sin\theta}$

$= \boxed{\cos\theta + \cot\theta}$ ✓

24. $(\csc\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}\,\dfrac{(1-\cos\theta)}{(1-\cos\theta)}$ mult. by $(1-\cos\theta)$

$= \dfrac{(1-\cos\theta)^2}{1-\cos^2\theta}$

$= \dfrac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \csc^2\theta(1-\cos\theta)(1-\cos\theta)$

$= \csc\theta(1-\cos\theta)\ \csc\theta(1-\cos\theta)$

$= \left(\csc\theta - \dfrac{\cos\theta}{\sin\theta}\right)\left(\csc\theta - \dfrac{\cos\theta}{\sin\theta}\right)$

$= \boxed{(\csc\theta - \cot\theta)^2}$ ✓

25. $\dfrac{1+\tan^2\theta}{1-\tan^2\theta} = \dfrac{1}{2\cos^2\theta - 1}$

$= \dfrac{\sec^2\theta}{1 - \frac{\sin^2\theta}{\cos^2\theta}}$

mult. by $\cos^2\theta$ $\rightarrow$

$= \dfrac{\sec^2\theta}{\frac{\cos^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}}$

$= \dfrac{1/\cos^2\theta}{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}$

$= \dfrac{1}{\cos^2\theta} \cdot \dfrac{\cos^2\theta}{\cos^2\theta - \sin^2\theta}$

$= \dfrac{1}{\cos^2\theta - \sin^2\theta}$

$= \dfrac{1}{\cos^2\theta - (1 - \cos^2\theta)}$

$= \dfrac{1}{\cos^2\theta - 1 + \cos^2\theta}$

$= \boxed{\dfrac{1}{2\cos^2\theta - 1}}$ ✓

26. $\tan^2\theta \cos^2\theta = 1 - \cos^2\theta$

$= \frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta$

$= \sin^2\theta$

$= \boxed{1 - \cos^2\theta}$ ✓

27. $\sec\theta - \cos\theta = \tan\theta \sin\theta$

$= \frac{1}{\cos\theta} - \cos\theta \frac{(\cos\theta)}{(\cos\theta)}$

$= \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}$

$= \frac{1 - \cos^2\theta}{\cos\theta}$

$= \frac{\sin^2\theta}{\cos\theta}$

$= \frac{\sin\theta \sin\theta}{\cos\theta}$

$= \boxed{\tan\theta \sin\theta}$ ✓

28. $1 - \tan^4\theta = 2\sec^2\theta - \sec^4\theta$

$= \sec^2\theta(2 - \sec^2\theta)$

$= (1 + \tan^2\theta)(2 - (1 + \tan^2\theta))$

$= (1 + \tan^2\theta)(2 - 1 - \tan^2\theta)$

$= (1 + \tan^2\theta)(1 - \tan^2\theta)$

$= \boxed{1 - \tan^4\theta}$ ✓

29. $(\csc\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta} \quad \frac{(1-\cos\theta)}{(1-\cos\theta)}$

$= \frac{1-2\cos\theta+\cos^2\theta}{1-\cos^2\theta}$

$= \frac{1-2\cos\theta+\cos^2\theta}{\sin^2\theta}$

$= \frac{1}{\sin^2\theta} - \frac{2\cos\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta}$

$= \csc^2\theta - \frac{2\cos\theta}{\sin\theta} \cdot \frac{1}{\sin\theta} + \cot^2\theta$

$= \csc^2\theta - 2\cot\theta\csc\theta + \cot^2\theta$

$\boxed{= (\csc\theta - \cot\theta)^2}$ ✓

30. $\frac{1+\tan\theta}{\sin\theta+\cos\theta} = \sec\theta$

mult. by $\cos\theta$ →

$= \frac{1+\frac{\sin\theta}{\cos\theta}}{\sin\theta+\cos\theta}$

$= \frac{\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\sin\theta+\cos\theta}$

$= \frac{\cos\theta+\sin\theta}{\cos\theta} \cdot \frac{1}{\sin\theta+\cos\theta}$

$= \frac{1}{\cos\theta}$

$= \boxed{\sec\theta}$ ✓

31. $\dfrac{2+\csc\theta\sec\theta}{\csc\theta\sec\theta} = (\sin\theta+\cos\theta)^2$

$= \dfrac{2}{\csc\theta\sec\theta} + \dfrac{\csc\theta\sec\theta}{\csc\theta\sec\theta}$

$= 2\sin\theta\cos\theta + 1$

$= 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta$

$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$= \boxed{(\sin\theta+\cos\theta)^2}$ ✓

44. $\sec^2\theta + \tan^2\theta = \sec^4\theta - \tan^4\theta$

$= (\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta)$

$= 1\,(\sec^2\theta + \tan^2\theta)$

$= \boxed{\sec^2\theta + \tan^2\theta}$ ✓

45. $-2\cos^2\theta = \sin^4\theta - \cos^4\theta - 1$

$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta) - 1$

$= 1\cdot(1 - \cos^2\theta - \cos^2\theta) - 1$

$= \boxed{-2\cos^2\theta}$ ✓

46. $\sec^2\theta \sin^2\theta = \sec^4\theta - (\tan^4\theta + \sec^2\theta)$

$= (\sec^4\theta - \tan^4\theta) - \sec^2\theta$

$= (\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta) - \sec^2\theta$

$= 1(\sec^2\theta + \sec^2\theta - 1) - \sec^2\theta$

$= \sec^2\theta + \sec^2\theta - 1 - \sec^2\theta$

$= \sec^2\theta - 1$

$= \tan^2\theta$

$= \frac{\sin^2\theta}{\cos^2\theta}$

$= \boxed{\sec^2\theta \sin^2\theta}$ ✓

47. $3\sec^2\theta\tan^2\theta + 1 = \sec^6\theta - \tan^6\theta$

$= (\sec^3\theta - \tan^3\theta)(\sec^3\theta + \tan^3\theta)$

$= (\sec\theta - \tan\theta)(\sec^2\theta + \sec\theta\tan\theta + \tan^2\theta) \cdot (\sec\theta + \tan\theta)(\sec^2\theta - \sec\theta\tan\theta + \tan^2\theta)$

* multiply binomials together * multiply trinomials together

$= (\sec^2\theta - \tan^2\theta)(\sec^4\theta - \sec^3\theta\tan\theta + \sec^2\theta\tan^2\theta + \sec^3\theta\tan\theta - \sec^2\theta\tan^2\theta + \sec\theta\tan^3\theta + \sec^2\theta\tan^2\theta - \sec\theta\tan^3\theta + \tan^4\theta)$

$= 1(\sec^4\theta + \sec^2\theta\tan^2\theta + \tan^4\theta)$

$= (\sec^4\theta + \sec^2\theta\tan^2\theta + \tan^4\theta)$

$= \sec^2\theta(1 + \tan^2\theta) + \tan^4\theta + \sec^2\theta\tan^2\theta$

$= \sec^2\theta + \sec^2\theta\tan^2\theta + \tan^4\theta + \sec^2\theta\tan^2\theta$

$= \sec^2\theta + 2\sec^2\theta\tan^2\theta + \tan^2\theta\tan^2\theta$

$= \sec^2\theta + 2\sec^2\theta\tan^2\theta + \tan^2\theta(\sec^2\theta - 1)$

$= \sec^2\theta + 2\sec^2\theta\tan^2\theta + \sec^2\theta\tan^2\theta - \tan^2\theta$

$= 3\sec^2\theta\tan^2\theta + 1$ ✓

48. $\sec^4 x = 1 + 2\tan^2 x + \tan^4 x$

$= (\tan^2 x + 1)^2$

$= (\sec^2 x)^2$

$= \boxed{\sec^4 x}$ ✓

49. $\sec^2 x \csc^2 x = \sec^2 x + \csc^2 x$

mult. by $\sin^2 x$ → $= \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$ ← mult. by $\cos^2 x$

$= \frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x}$

$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}$

$= \frac{1}{\sin^2 x \cos^2 x}$

$= \boxed{\sec^2 x \csc^2 x}$ ✓