

## Trig Identities 5-1

22.  $\csc x \sec x - \tan x$

$$\frac{1}{\sin x} \cdot \frac{1}{\cos x} - \frac{\sin x}{\cos x} \begin{matrix} (\sin x) \\ (\sin x) \\ \leftarrow \text{mult. by } \sin x \end{matrix}$$

$$\frac{1}{\sin x \cos x} - \frac{\sin^2 x}{\sin x \cos x}$$

$$\frac{1 - \sin^2 x}{\sin x \cos x}$$

$$\frac{\cos^2 x}{\sin x \cos x}$$

$$\frac{\cos x}{\sin x}$$

$$\frac{\cos x}{\sin x} = \boxed{\cot x}$$

23.  $\csc x - \cos x \cot x$

$$\frac{1}{\sin x} - \cos x \cdot \frac{\cos x}{\sin x}$$

$$\frac{1 - \cos^2 x}{\sin x}$$

$$\frac{\sin^2 x}{\sin x} = \boxed{\sin x}$$

24.  $\sec x \cot x - \sin x$

$$\frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} - \sin x \begin{matrix} (\sin x) \\ (\sin x) \end{matrix}$$

$$\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$$

$$\frac{1 - \sin^2 x}{\sin x}$$

$$\frac{\cos^2 x}{\sin x}$$

25.

$$\frac{\tan x + \sin x \sec x}{\csc x \tan x}$$

$$\frac{\frac{\sin x}{\cos x} + \frac{\sin x}{1} \cdot \frac{1}{\cos x}}{\csc x \tan x}$$

$$\frac{\frac{2 \sin x}{\cos x}}{\csc x \tan x}$$

$$\frac{2 \tan x}{\csc x \tan x}$$

$$\frac{2}{\csc x} = \boxed{2 \sin x}$$

26.

$$\frac{1 - \sin^2 x}{\csc^2 x - 1}$$

$$\frac{\cos^2 x}{\cot^2 x}$$

$$\frac{\cos^2 x}{\cos^2 x / \sin^2 x}$$

$$\cos^2 x \cdot \frac{\sin^2 x}{\cos^2 x} = \boxed{\sin^2 x}$$

27.

$$\frac{\csc x \cos x + \cot x}{\sec x \cot x}$$

$$\frac{1}{\sin x} \cdot \cos x + \cot x$$

$$\sec x \cot x$$

$$\frac{\cos x}{\sin x} + \cot x$$

$$\sec x \cot x$$

$$\frac{\cot x + \cot x}{\sec x \cot x}$$

$$\frac{2\cot x}{\sec x \cot x} = \frac{2}{\sec x} = \boxed{2\cos x}$$

28.

$$\frac{\sec x \csc x - \tan x}{\sec x \csc x}$$

$$\frac{\frac{1}{\cos x} \cdot \frac{1}{\sin x} - \frac{\sin x}{\cos x} \left(\frac{\sin x}{\sin x}\right)}{\sec x \csc x} \quad \leftarrow \text{mult. by } \sin x$$

$$\frac{\frac{1}{\cos x \sin x} - \frac{\sin^2 x}{\cos x \sin x}}{\sec x \csc x}$$

$$\frac{1 - \sin^2 x}{\cos x \sin x}$$

$$\sec x \csc x$$

$$\frac{\cos^2 x}{\cos x \sin x}$$

$$\frac{1}{\cos x} \cdot \frac{1}{\sin x}$$

$$\frac{\cos^2 x}{\cos x \sin x}$$

$$\frac{\cos x}{\cancel{\sin x}} \cdot \frac{\cos x \cancel{\sin x}}{1} = \boxed{\cos^2 x}$$

29.

$$\frac{\sec^2 x}{\cot^2 x + 1}$$

$$\frac{\sec^2 x}{\csc^2 x}$$

$$\frac{1/\cos^2 x}{1/\sin^2 x}$$

$$\frac{1}{\cos^2 x} \cdot \frac{\sin^2 x}{1} = \frac{\sin^2 x}{\cos^2 x} = \boxed{\tan^2 x}$$

30.

$$\cot x - \csc^2 x \cot x$$

$$\text{factor out } \cot x \rightarrow \cot x (1 - \csc^2 x)$$

$$\cot x (-\cot^2 x)$$

$$\boxed{-\cot^3 x}$$

31.

$$\cot x - \cos^3 x \csc x$$

$$\frac{\cos x}{\sin x} - \frac{\cos^3 x}{\sin x}$$

$$\frac{\cos x - \cos^3 x}{\sin x}$$

$$\frac{\cos x (1 - \cos^2 x)}{\sin x}$$

$$\frac{\cos x (\sin^2 x)}{\sin x} = \boxed{\cos x \sin x}$$

32.

$$\frac{\cos x}{\sec x + 1} + \frac{\cos x}{\sec x - 1}$$

$$\text{mult. by conjugate } \rightarrow \frac{\cos x (\sec x - 1)}{(\sec x + 1)(\sec x - 1)} + \frac{\cos x (\sec x + 1)}{(\sec x + 1)(\sec x - 1)}$$

33.

mult. by  
 $1 + \cos x$ 

$$\rightarrow \frac{1 - \cos x}{\tan x} + \frac{\sin x}{1 + \cos x} \leftarrow \text{mult. by } \tan x$$

$$\frac{(1 - \cos x)(1 + \cos x)}{\tan x (1 + \cos x)} + \frac{\sin x \tan x}{\tan x (1 + \cos x)}$$

$$\frac{1 - \cos^2 x + \sin x \tan x}{\tan x (1 + \cos x)}$$

$$\frac{\sin^2 x + \sin x \tan x}{\tan (1 + \cos x)}$$

$$\frac{\sin^2 x + \sin x \tan x}{\tan x + \cos x \tan x}$$

$$\frac{\sin^2 x + \sin x \tan x}{\tan x + \cos x \cdot \frac{\sin x}{\cos x}}$$

$$\frac{\sin x (\sin x + \tan x)}{\cancel{\tan x} + \sin x} = \boxed{\sin x}$$

34.

$$\frac{1}{\sec x + 1} + \frac{1}{\sec x - 1} \leftarrow \text{multiply by conjugates } (\sec x - 1), (\sec x + 1)$$

$$\frac{\sec x - 1}{(\sec x + 1)(\sec x - 1)} + \frac{\sec x + 1}{(\sec x - 1)(\sec x + 1)}$$

$$\frac{\sec x - 1 + \sec x + 1}{\sec^2 x - 1}$$

$$\frac{2 \sec x}{\tan^2 x}$$

$$\frac{2(1/\cos x)}{\sin^2 x / \cos^2 x}$$

$$\frac{2}{\cos^2 x} = \frac{2 \cos x}{\cos x} = 2 \cdot \cos x \cdot \frac{1}{\cos x} = \boxed{2 \cot x \csc x}$$

35.

$$\frac{\cos x \cot x}{\sec x + \tan x} + \frac{\sin x}{\sec x - \tan x} \quad \text{multiply by conjugates } (\sec x - \tan x), (\sec x + \tan x)$$

$$= \frac{\cos x \cot x (\sec x - \tan x)}{(\sec x + \tan x)(\sec x - \tan x)} + \frac{\sin x (\sec x + \tan x)}{(\sec x - \tan x)(\sec x + \tan x)}$$

$$= \frac{\cos x \cot x \sec x - \cos x \cot x \tan x + \sin x \sec x + \sin x \tan x}{\sec^2 x - \tan^2 x}$$

$$= \frac{\cancel{\cos x} \cot x \cdot \frac{1}{\cancel{\cos x}} - \cancel{\cos x} \cdot \frac{1}{\cancel{\cos x}} \cdot \cancel{\tan x} + \sin x \cdot \frac{1}{\cancel{\cos x}} + \sin x \tan x}{1}$$

$$= \boxed{\cot x - \cos x + \tan x + \sin x \tan x}$$

36.

$$\frac{\sin x}{\csc x + 1} + \frac{\sin x}{\csc x - 1} \quad \text{multiply by conjugates } (\csc x - 1), (\csc x + 1)$$

$$= \frac{\sin x (\csc x - 1)}{(\csc x + 1)(\csc x - 1)} + \frac{\sin x (\csc x + 1)}{(\csc x - 1)(\csc x + 1)}$$

$$= \frac{\sin x \csc x - \sin x + \sin x \csc x + \sin x}{\csc^2 x - 1}$$

$$= \frac{2 \sin x \csc x}{\csc^2 x - 1}$$

$$= \frac{2 \sin \left( \frac{1}{\sin x} \right)}{\cot^2 x}$$

$$= \frac{2}{\cot^2 x}$$

$$= \frac{2}{1/\tan^2 x}$$

38,

$$\frac{\sin x}{\csc x - \cot x}$$

$$= \frac{\sin x}{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}$$

$$= \frac{\sin x}{\frac{1 - \cos x}{\sin x}}$$

$$= \sin x \cdot \frac{\sin x}{1 - \cos x}$$

$$= \frac{\sin^2 x}{1 - \cos x}$$

$$= \frac{1 - \cos^2 x}{1 - \cos x} = \frac{(1 - \cos x)(1 + \cos x)}{1 - \cos x} = \boxed{1 + \cos x}$$

39,

$$\frac{\csc x}{1 - \sin x} \quad \text{multiply by conjugate}$$

$$\frac{\csc x (1 + \sin x)}{(1 - \sin x)(1 + \sin x)}$$

$$\frac{\csc x + \csc x \sin x}{1 - \sin^2 x}$$

$$\frac{\csc x + \frac{1}{\sin x} \cdot \sin x}{1 - \sin^2 x}$$

40.

$$\frac{\cot x}{\sec x - \tan x}$$

multiply by the conjugate  
 $\sec x + \tan x$

$$= \frac{\cot x (\sec x + \tan x)}{(\sec x - \tan x)(\sec x + \tan x)}$$

$$= \frac{\cot x \sec x + \cot x \tan x}{\sec^2 x - \tan^2 x}$$

$$= \frac{\frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} + \frac{\cos x}{\sin x} \cdot \frac{\sin x}{\cos x}}{1}$$

$$= \frac{1}{\sin x} + 1$$

$$\boxed{\csc x + 1}$$

41.

$$\frac{\cot x}{1 + \sin x} \quad (1 - \sin x) \text{ multiply by conjugate } 1 - \sin x$$

$$= \frac{\cot x (1 - \sin x)}{1 - \sin^2 x}$$

$$= \frac{\cot x (1 - \sin x)}{\cos^2 x}$$

$$= \frac{\cot x}{\cos^2 x} (1 - \sin x)$$

$$= \frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x} \cdot (1 - \sin x)$$



$$42. \quad \frac{3 \tan x (1 + \cos x)}{1 - \cos x (1 + \cos x)} \quad \leftarrow \text{multiply by } 1 + \cos x$$

$$= \frac{3 \tan x (1 + \cos x)}{1 - \cos^2 x}$$

$$= \frac{3 \sin x}{\cos x} \cdot \frac{(1 + \cos x)}{\sin^2 x}$$

$$= \frac{3 \frac{\sin x}{\cos x} + 3 \sin x}{\sin^2 x}$$

$$= \frac{3 \sin x \left( \frac{1}{\cos x} + 1 \right)}{\sin^2 x}$$

$$= \frac{3 \sin x (\sec x + 1)}{\sin^2 x}$$

$$= 3 (\sec x + 1) \cdot \frac{1}{\sin x}$$

$$= \boxed{3 \csc x (\sec x + 1)}$$

$$43. \quad \frac{2 \sin x (\cot x - \csc x)}{\cot x + \csc x (\cot x - \csc x)} \quad \text{Mult. by conjugate } (\cot x - \csc x)$$

$$= \frac{2 \sin x (\cot x - \csc x)}{\cot^2 x - \csc^2 x}$$

$$= \frac{2 \sin x \left( \frac{\cos x}{\sin x} - \frac{1}{\sin x} \right)}{-1}$$

$$= \frac{2 \cos x - 2}{-1}$$

$$= \boxed{-2 \cos x + 2}$$

44,

$$\frac{\sin x (1 + \sec x)}{1 - \sec x (1 + \sec x)}$$

mult. by conjugate  $(1 + \sec x)$ 

$$= \frac{\sin x (1 + \sec x)}{1 - \sec^2 x}$$

$$= \frac{\sin x \overbrace{(1 + \sec x)}^{\cancel{1 + \sec x}}}{-\tan^2 x}$$

$$= \frac{\sin x + \sin x \sec x}{-\tan^2 x}$$

$$= \frac{\sin x}{-\tan^2 x} + \frac{\sin x \sec x}{-\tan^2 x}$$

$$= \frac{\sin x}{-\sin^2 x / \cos^2 x} + \frac{\sin x \sec x}{-\sin^2 x / \cos^2 x}$$

$$= \sin x \cdot -\frac{\cos^2 x}{\sin^2 x} + \sin x \cdot \frac{1}{\cos x} \cdot \frac{\cos^2 x}{-\sin^2 x}$$

$$= -\frac{\cos^2 x}{\sin x} - \frac{\cos x}{\sin x}$$

$$= \frac{-\cos^2 x - \cos x}{\sin x}$$

$$= \frac{-\cos x (\cos x + 1)}{\sin x}$$

$$= \boxed{-\cot x (\cos x + 1)}$$

$$45. \frac{\cot^2 x \cos x}{\csc x - 1} \quad \begin{array}{l} \text{multiply by } \csc x + 1 \\ (\csc x + 1) \end{array}$$

$$= \frac{\cot^2 x \cos x (\csc x + 1)}{\csc^2 x - 1}$$

$$= \frac{\cot^2 x \cos x (\csc x + 1)}{\cot^2 x}$$

$$= \cos \left( \frac{1}{\sin x} + 1 \right) = \frac{\cos x}{\sin x} + \cos x = \boxed{\cot x + \cos x}$$

$$46. \frac{5 (\sec x - 1)}{\sec x + 1} \quad \text{mult. by } \sec x - 1$$

$$= \frac{5 (\sec x - 1)}{\sec^2 x - 1}$$

$$= \frac{5 (\sec x - 1)}{\tan^2 x}$$

$$= \boxed{5 \cot^2 x (\sec x - 1)}$$

$$47. \frac{\sin x \tan x (\cos x - 1)}{\cos x + 1} \quad \text{mult. by } \cos x - 1$$

$$= \frac{\sin x \tan x (\cos x - 1)}{\cos^2 x - 1}$$

$$= \frac{\sin x \cdot \frac{\sin x}{\cos x} (\cos x - 1)}{-\sin^2 x}$$

$$= \frac{\sin^2 x \cdot \frac{1}{\cos x} (\cos x - 1)}{-\sin^2 x}$$

$$= -\frac{1}{\cos x} (\cos x - 1)$$

$$= -1 + \frac{1}{\cos x}$$

$$51. \quad \tan x - \csc x \sec x$$

$$= \frac{\sin x}{\cos x} - \frac{1}{\sin x} \cdot \frac{1}{\cos x}$$

$$= \frac{\sin x (\sin x)}{\cos x (\sin x)} - \frac{1}{\sin x \cos x}$$

$$= \frac{\sin^2 x}{\sin x \cos x} - \frac{1}{\sin x \cos x}$$

$$= \frac{\sin^2 x - 1}{\sin x \cos x}$$

$$= \frac{-\cos^2 x}{\sin x \cos x} = -\frac{\cos x}{\sin x} = \boxed{-\cot x}$$

$$52. \quad \cos x + \tan x \sin x$$

$$= \cos x + \frac{\sin x}{\cos x} \cdot \sin x$$

$$= \cos x + \frac{\sin^2 x}{\cos x}$$

mult. by  $\cos \rightarrow$

$$= \frac{\cos x (\cos x)}{(\cos x)} + \frac{\sin^2 x}{\cos x}$$

$$= \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos x}$$

$$= \frac{1}{\cos x}$$

$$= \boxed{\sec x}$$

$$53. \quad \csc x \tan^2 x - \sec^2 x \csc x$$

$$\csc x (\tan^2 x - \sec^2 x)$$

$$\csc x (-1)$$

$$\boxed{-\csc x}$$

$$54. \quad \sec x \csc x - \cos x \csc x$$

$$\csc x (\sec x - \cos x)$$

$$\csc x \left( \frac{1}{\cos x} - \cos x \right) \leftarrow \text{mult. by } \cos x$$

$$\csc x \left( \frac{1 - \cos^2 x}{\cos x} \right)$$

$$\frac{1}{\sin x} \cdot \frac{\sin^2 x}{\cos x}$$

$$\frac{\sin x}{\cos x}$$

$$\boxed{\tan x}$$