Verifying Identities WS

1. $\cos^2 A \csc A \sec A = \cot A$

$\cos^2 A \cdot \frac{1}{\sin A} \cdot \frac{1}{\cos A}$

$\frac{\cos^2 A}{\sin A \cos A} = \frac{\cos A}{\sin A} = \cot A$ ✓

2. $\tan\beta(\sin\beta + \cot\beta\cos\beta) = \sec\beta$

$\frac{\sin\beta}{\cos\beta}\left(\sin\beta + \frac{\cos\beta}{\sin\beta}\cos\beta\right)$

$\frac{\sin^2\beta}{\cos\beta} + \frac{\cos^2\beta \sin\beta}{\cos\beta \sin\beta}$

$\frac{\sin^2\beta}{\cos\beta} + \frac{\cos^2\beta}{\cos\beta}$

$\frac{\sin^2\beta + \cos^2\beta}{\cos\beta} = \frac{1}{\cos\beta} = \sec\beta$ ✓

3. $\cos x(\sec x + \cos x \csc^2 x) = \csc^2 x$

$\cos x\left(\frac{1}{\cos x} + \cos x \cdot \frac{1}{\sin^2 x}\right)$

$\cos x\left(\frac{1}{\cos x} + \frac{\cos x}{\sin^2 x}\right)$

$\frac{\cos x}{\cos x} + \frac{\cos^2 x}{\sin^2 x} = 1 + \cot^2 x = \csc^2 x$ ✓

4. $(\cos x - \sin x)^2 = 1 - 2\sin x \cos x$

$(\cos x - \sin x)(\cos x - \sin x)$

$\cos^2 x - 2\sin x \cos x + \sin^2 x$

$\cos^2 x + \sin^2 x - 2\sin x \cos x$

$1 - 2\sin x \cos x$ ✓

5. $(\tan\beta + \cot\beta)^2 = \sec^2\beta + \csc^2\beta$

$(\tan\beta + \cot\beta)(\tan\beta + \cot\beta)$

$\tan^2\beta + 2\cot\beta\tan\beta + \cot^2\beta$

$\tan^2\beta + 2(1) + \cot^2\beta$

$\sec^2\beta - 1 + 2 + \cot^2\beta$

$\sec^2\beta + 1 + \cot^2\beta$

$\sec^2\beta + \csc^2\beta$ ✓

6. $\dfrac{1 + \cot^2 X}{\sec^2 X} = \cot^2 X$

$\dfrac{\csc^2 X}{\sec^2 X}$

$\dfrac{1/\sin^2 X}{1/\cos^2 X} = \dfrac{1}{\sin^2 X} \cdot \dfrac{\cos^2 X}{1} = \dfrac{\cos^2 X}{\sin^2 X} = \cot^2 X$ ✓

7. $\frac{\cos A}{\cos A}\frac{\sec A}{\sin A} - \frac{\sin A}{\cos A}\frac{(\sin A)}{(\sin A)}\cot A$

$$\frac{\cos A \sec A}{\cos A \sin A} - \frac{\sin^2 A}{\cos A \sin A}$$

$$\frac{\cos A\,(1/\cos A) - \sin^2 A}{\cos A \sin A}$$

$$\frac{1-\sin^2 A}{\cos A \sin A} = \frac{\cos^2 A}{\cos A \sin A} = \frac{\cos A}{\sin A} = \cot A \checkmark$$

8. $$\frac{1}{1-\cos y} + \frac{1}{1+\cos y} = 2\csc^2 y$$

* multiply by conjugates

$$\frac{1}{1-\cos y}\frac{(1+\cos y)}{(1+\cos y)} + \frac{1}{1+\cos y}\frac{(1-\cos y)}{(1-\cos y)}$$

$$\frac{1+\cos y}{1-\cos^2 y} + \frac{1-\cos y}{1-\cos^2 y}$$

$$\frac{1+\cos y + 1-\cos y}{1-\cos^2 y} = \frac{2}{\sin^2 y} = 2\csc^2 y \checkmark$$

9. $\cot^2 x \csc^2 x - \cot^2 x = \cot^4 x$

$\cot^2 x (\csc^2 x - 1)$

$\cot^2 x (\cot^2 x)$

$\cot^4 x$ ✓

10. $\sec^4 a - \tan^4 a = 1 + 2\tan^2 a$

$(\sec^2 a - \tan^2 a)(\sec^2 a + \tan^2 a)$

$(1)\ (\sec^2 a + \tan^2 a)$

$\sec^2 a + \tan^2 a$

$\tan^2 a + 1 + \tan^2 a$

$1 + 2\tan^2 a$ ✓

11. $\frac{1}{\sin x \cos x} - \frac{\cos x}{\sin x (\cos x)} = \tan x$

$\frac{1}{\sin x \cos x} - \frac{\cos^2 x}{\sin x \cos x}$

$\frac{1 - \cos^2 x}{\sin x \cos x} = \frac{\sin^2 x}{\sin x \cos x} = \frac{\sin x}{\cos x} = \tan x$ ✓

12. $\frac{1}{1 - \sin r} = \sec^2 r + \sec r \tan r$

↖ multiply by conjugate

$\frac{1}{1 - \sin r} \cdot \frac{1 + \sin r}{1 + \sin r}$

$\frac{1 + \sin r}{1 - \sin^2 r} = \frac{1 + \sin r}{\cos^2 r} = \frac{1}{\cos^2 r} + \frac{\sin r}{\cos^2 r} = \sec^2 r + \left(\frac{\sin r}{\cos r} \cdot \frac{1}{\cos r}\right)$

$= \sec^2 r + \tan r \sec r = \sec^2 r + \sec r \tan r$ ✓

13. $\frac{\cos x}{\sec x - 1} - \frac{\cos x}{\tan^2 x} = \cot^2 x$

↖ multiply by conjugate

$\frac{\cos x(\sec x + 1)}{\sec x - 1(\sec x + 1)} - \frac{\cos x}{\tan^2 x}$

$\frac{\cos x\left(\frac{1}{\cos x} + 1\right)}{\sec^2 x - 1} - \frac{\cos x}{\tan^2 x}$

$\frac{1 + \cos x}{\tan^2 x} - \frac{\cos x}{\tan^2 x} = \frac{1 + \cos x - \cos x}{\tan^2 x} = \frac{1}{\tan^2 x} = \cot^2 x$ ✓

14. $\frac{\sec x}{\sec x - \tan x} = \sec^2 x + \sec x \tan x$

↖ multiply by conjugate

$\frac{\sec x(\sec x + \tan x)}{(\sec x - \tan x)(\sec x + \tan x)}$

$\frac{\sec^2 x + \sec x \tan x}{\sec^2 x - \tan^2 x}$

$\frac{\sec^2 x + \sec x \tan x}{1}$

$\sec^2 x + \sec x \tan x$ ✓

15. $\frac{1+\sin x}{1-\sin x} = 2\sec^2 x + 2\sec x \tan x - 1$

↖ multiply by conjugate

$\frac{1+\sin x}{1-\sin x}\cdot\frac{(1+\sin x)}{(1+\sin x)}$

$\frac{1+2\sin x+\sin^2 x}{1-\sin^2 x}$

$\frac{1+2\sin x+\sin^2 x}{\cos^2 x}$

$\frac{1}{\cos^2 x} + \frac{2\sin x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$

$\sec^2 x + \frac{2\sin x}{\cos x}\cdot\frac{1}{\cos x} + \tan^2 x$

$\sec^2 x + 2\tan x \cdot \sec x + \tan^2 x$

$\sec^2 x + 2\sec x \tan x + \sec^2 x - 1$

$2\sec^2 x + 2\sec x \tan x - 1$ ✓

16. $\sin^3 y \cos^2 y = \sin^3 y - \sin^5 y$

$= \sin^3 y (1 - \sin^2 y)$

$= \sin^3 y \cos^2 y$ ✓

17. $\sec^2\theta + \csc^2\theta = \sec^2\theta \csc^2\theta$

$\frac{(\sin^2\theta)}{(\sin^2\theta)} \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} \frac{(\cos^2\theta)}{(\cos^2\theta)}$

$\frac{\sin^2\theta}{\sin^2\theta\cos^2\theta} + \frac{\cos^2\theta}{\sin^2\theta\cos^2\theta}$

$\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta} = \frac{1}{\sin^2\theta\cos^2\theta} = \sec^2\theta \csc^2\theta$ ✓

18. $\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta} \frac{(\sec\theta + \tan\theta)}{(\sec\theta + \tan\theta)}$

↖ multiply by conjugate

$= \frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$

$= \frac{\sec\theta + \tan\theta}{1}$

$= \sec\theta + \tan\theta$ ✓

19. $\frac{1 - 3\cos x - 4\cos^2 x}{\sin^2 x} = \frac{1 - 4\cos x}{1 - \cos x} \frac{(1 + \cos x)}{(1 + \cos x)}$ ← multiply by conjugate

$= \frac{1 - 4\cos x + \cos x - 4\cos^2 x}{1 - \cos^2 x}$

$= \frac{1 - 3\cos x - 4\cos^2 x}{\sin^2 x}$ ✓

20. $\dfrac{\sec^2 x - 6\tan x + 7}{\sec^2 x - 5} = \dfrac{\tan x - 4}{\tan x + 2} \quad \dfrac{(\tan x - 2)}{(\tan x - 2)}$

mult. by conjugate

$$= \frac{(\tan x - 4)(\tan x - 2)}{(\tan x + 2)(\tan x - 2)}$$

$$= \frac{\tan^2 x - 6\tan x + 8}{\tan^2 x - 4}$$

$$= \frac{\sec^2 x - 1 - 6\tan x + 8}{\sec^2 x - 1 - 4}$$

$$= \frac{\sec^2 x - 6\tan x + 7}{\sec^2 x - 5} \quad \checkmark$$

21. $\dfrac{\sec^3 x - \cos^3 x}{\sec x - \cos x} = \sec^2 x + 1 + \cos^2 x$

mult. by conjugate →

$$\frac{(\sec^3 x - \cos^3 x)(\sec x + \cos x)}{(\sec x - \cos x)(\sec x + \cos x)}$$

regroup →

$$\frac{\sec^4 x + \cos x \sec^3 x - \cos^3 x \sec x - \cos^4 x}{\sec^2 x - \cos^2 x}$$

$$\frac{(\sec^4 x - \cos^4 x) + (\cos x \sec^3 x - \cos^3 x \sec x)}{\sec^2 x - \cos^2 x}$$

Factor out →

$$\frac{(\sec^2 x - \cos^2 x)(\sec^2 x + \cos^2 x) + \cos x \sec x (\sec^2 x - \cos^2 x)}{\sec^2 x - \cos^2 x}$$

$$\frac{(\sec^2 x - \cos^2 x)\left((\sec^2 x + \cos^2 x) + (\cos x \sec x)\right)}{\sec^2 x - \cos^2 x}$$

$$\sec^2 x + \cos^2 x + \cos x \sec x$$

$$\sec^2 x + \cos^2 x + 1$$

$$\sec^2 x + 1 + \cos^2 x \quad \checkmark$$

22. $(2\sin x + 3\cos x)^2 + (3\sin x - 2\cos x)^2 = 13$

$4\sin^2 x + 12\sin x\cos x + 9\cos^2 x + 9\sin^2 x - 12\sin x\cos x + 4\cos^2 x$

$4\sin^2 x + 4\cos^2 x + 9\cos^2 x + 9\sin^2 x$

$4(\sin^2 x + \cos^2 x) + 9(\cos^2 x + \sin^2 x)$

$4(1) + 9(1)$

13 ✓

23. $\dfrac{1+\sin x+\cos x}{1+\sin x-\cos x} = \dfrac{1+\cos x}{\sin x}$

$\dfrac{(1+\sin x+\cos x)(1-\sin x+\cos x)}{(1+\sin x-\cos x)(1-\sin x+\cos x)}$

$\dfrac{1-\sin x+\cos x+\sin x-\sin^2 x+\cos x\sin x+\cos x-\cos x\sin x+\cos^2 x}{1-\sin x+\cos x+\sin x-\sin^2 x+\cos x\sin x-\cos x+\cos x\sin x-\cos^2 x}$

$\dfrac{1+2\cos x-\sin^2 x+\cos^2 x}{1-\sin^2 x-\cos^2 x+2\cos x\sin x}$

$\dfrac{1+2\cos x-(1-\cos^2 x)+\cos^2 x}{1-(\sin^2 x+\cos^2 x)+2\cos x\sin x}$

$\dfrac{1+2\cos x-1+\cos^2 x+\cos^2 x}{1-1+2\cos x\sin x}$

$\dfrac{2\cos x+2\cos^2 x}{2\cos x\sin x}$

$\dfrac{2\cos x(1+\cos^2 x)}{2\cos x\sin x}$

$\dfrac{1+\cos^2 x}{\sin x}$ ✓

24. $\dfrac{1+\sin x+\cos x}{1-\sin x+\cos x} = \dfrac{1+\sin x}{\cos x}$

$$\frac{(1+\sin x+\cos x)(1+\sin x-\cos x)}{(1-\sin x+\cos x)(1+\sin x-\cos x)}$$

$$\frac{1+\sin x-\cos x+\sin x+\sin^2 x-\cos x\sin x+\cos x+\cos x\sin x-\cos^2 x}{1+\sin x-\cos x-\sin x-\sin^2 x+\cos x\sin x+\cos x+\cos x\sin x-\cos^2 x}$$

$$\frac{1+2\sin x+\sin^2 x-\cos^2 x}{1-\sin^2 x-\cos^2 x+2\cos x\sin x}$$

(-1) ←

$$\frac{1+2\sin x+\sin^2 x-(1-\sin^2 x)}{1-1+2\cos x\sin x}$$

$$\frac{1+2\sin x+\sin^2 x-1+\sin^2 x}{2\cos x\sin x}$$

$$\frac{2\sin^2 x+2\sin x}{2\cos x\sin x}$$

$$\frac{2\sin x(\sin x+1)}{2\cos x\sin x}$$

$$\frac{\sin x+1}{\cos x}$$ ✓