

Sin Cos Graphs with all transformations

1. $f(x) = \sin x$ $g(x) = \sin(x - \pi)$ $\left. \begin{array}{l} x - \pi = 0 \\ x = \pi \end{array} \right\} \begin{array}{l} \text{Phase shift} \\ \text{right } \pi \end{array}$

2. $f(x) = \cos x$ $g(x) = \cos(x + \pi)$ $\left. \begin{array}{l} x + \pi = 0 \\ x = -\pi \end{array} \right\} \begin{array}{l} \text{Phase shift} \\ \text{left } \pi \end{array}$

3. $f(x) = \sin x$ $g(x) = 4 + \sin x$
 ↖ Vertical shift up 4

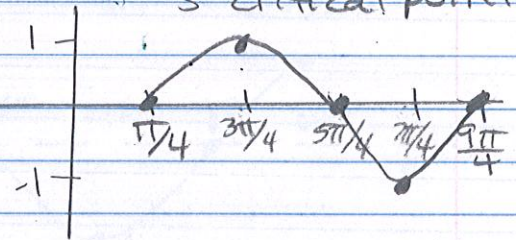
4. $f(x) = \cos x$ $g(x) = -6 + \cos 4x$
 ↖ Vertical shift down 6 ↖ Period $\frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$

5. $y = \sin(x - \frac{\pi}{4})$

Phase Shift

$\left\{ \begin{array}{l} x - \frac{\pi}{4} = 0 \\ x = \frac{\pi}{4} \end{array} \right.$ $x - \frac{\pi}{4} = 2\pi$
 $x = 2\pi + \frac{\pi}{4} = \frac{8\pi}{4} + \frac{\pi}{4} = \frac{9\pi}{4}$

5 critical points



Amp = 1

Period = $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

Domain = $[\frac{\pi}{4}, \frac{9\pi}{4}]$

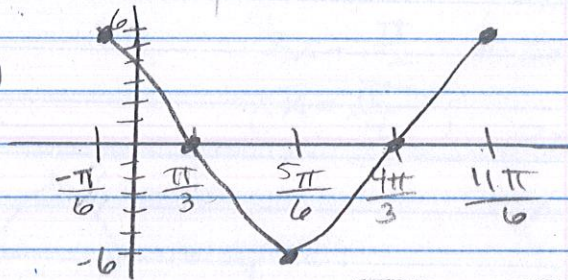
Range = $[-1, 1]$

* No vertical shift

6. $y = 6 \cos(x + \frac{\pi}{6})$

Amp = 6

Period = $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$



Phase Shift $x + \frac{\pi}{6} = 0$ $x + \frac{\pi}{6} = 2\pi$

$x = -\frac{\pi}{6}$

$x = \frac{12\pi}{6} - \frac{\pi}{6} = \frac{11\pi}{6}$

* Take average of start + end

$-\frac{\pi}{6} + \frac{11\pi}{6} = \frac{10\pi}{6} \cdot \frac{1}{2}$

$= \frac{5\pi}{6}$

Domain: $[-\frac{\pi}{6}, \frac{11\pi}{6}]$

Range: $[-6, 6]$

$\frac{5\pi}{6} + \frac{11\pi}{6} = \frac{16\pi}{6} = \frac{8\pi}{3} \cdot \frac{1}{2} = \frac{4\pi}{3}$

$-\frac{\pi}{6} + \frac{5\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{3}$

7. $y = 2 \sin(\theta + 90^\circ)$

Phase Shift

$\theta + 90^\circ = 0^\circ$ $\theta + 90^\circ = 360^\circ$

$\theta = -90^\circ$ $\theta = 270^\circ$

Amp = 2

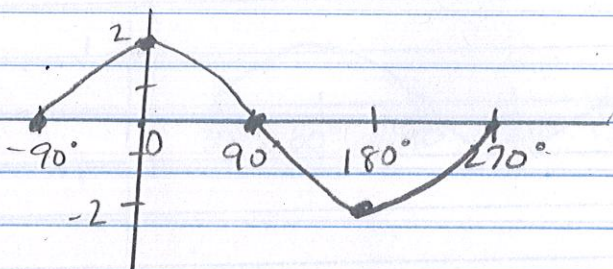
Period = 360°

PS = -90°

VS = none

D: $[-90^\circ, 270^\circ]$

R: $[-2, 2]$



8. $y = 7 \cos 3\theta - 2$

$3\theta = 0$ $3\theta = 360^\circ$

$\theta = 0^\circ$ $\theta = 120^\circ$

Amp = 7

Period = $\frac{360^\circ}{3} = 120^\circ$

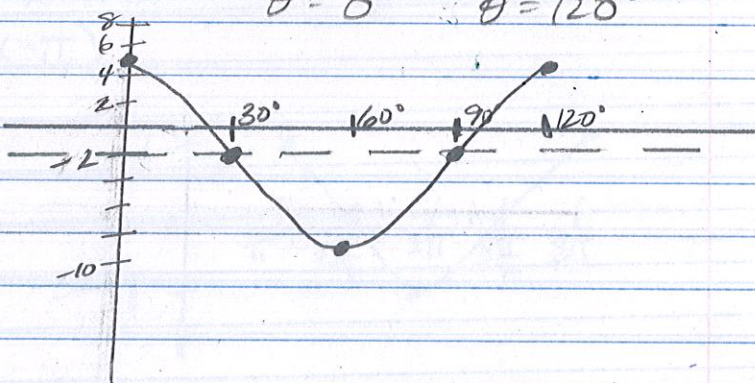
PS = None

VS = -2

D: $[0^\circ, 120^\circ]$

R: $[-7-2, 7-2]$

$\hookrightarrow [-9, 5]$



9. $y = 3 \sin(2x - \frac{\pi}{2}) + 2$

$2x - \frac{\pi}{2} = 0$

$2x - \frac{\pi}{2} = 2\pi$

$2x = \frac{\pi}{2}$

$2x = \frac{4\pi}{2} + \frac{\pi}{2}$

$x = \frac{\pi}{4}$

$2x = \frac{5\pi}{2}$

$x = \frac{5\pi}{2} \cdot \frac{1}{2}$

$x = \frac{5\pi}{4}$

Amp = 3

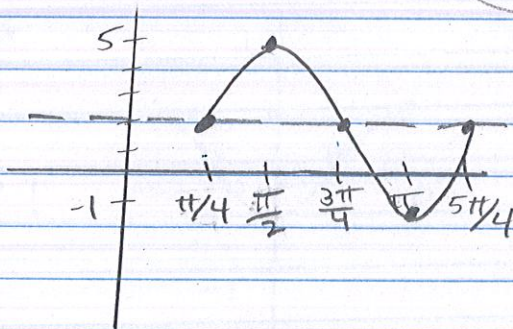
Period = $\frac{2\pi}{2} = \pi$

PS =

VS = 2

D =

R = $[-1, 5]$



$$10. \quad y = \sin(3\theta - 90^\circ)$$

$$3\theta - 90^\circ = 0$$

$$3\theta - 90^\circ = 360^\circ$$

$$3\theta = 90^\circ$$

$$3\theta = 450^\circ$$

$$\theta = 30^\circ$$

$$\theta = 150^\circ$$

$$\text{Amp} = 1$$

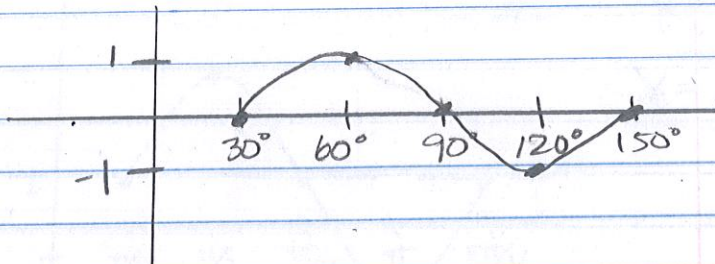
$$\text{Pd} = \frac{360}{3} = 120^\circ$$

$$\text{PS} = 30^\circ$$

$$\text{VS} = \text{none}$$

$$\text{D} = [30^\circ, 150^\circ]$$

$$\text{R} = [-1, 1]$$



* factor out negative ↘

$$11. \quad y = \cos(-x + \pi)$$

$$y = \cos(-(x - \pi))$$

$$y = \cos(x - \pi)$$

$$x - \pi = 0$$

$$x - \pi = 2\pi$$

$$x = \pi$$

$$x = 3\pi$$

$$\text{Amp} = 1$$

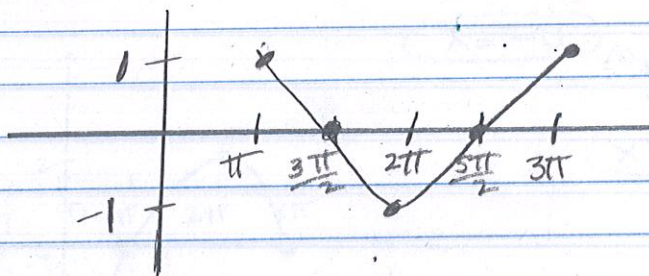
$$\text{Period} = \frac{2\pi}{1} = 2\pi$$

$$\text{PS} = \pi$$

$$\text{VS} = \text{none}$$

$$\text{D} = [\pi, 3\pi]$$

$$\text{R} = [-1, 1]$$



$$12. \quad y = \frac{1}{2} \cos(3\theta + 180^\circ)$$

$$3\theta + 180^\circ = 0$$

$$3\theta + 180^\circ = 360^\circ$$

$$3\theta = -180^\circ$$

$$3\theta = 180^\circ$$

$$\theta = -60^\circ$$

$$\theta = 60^\circ$$

$$\text{Amp} = \frac{1}{2}$$

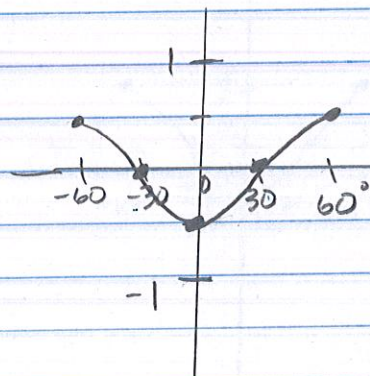
$$\text{pd} = \frac{360}{3} = 120^\circ$$

$$\text{PS} = -60^\circ$$

$$\text{VS} = \text{none}$$

$$\text{D} = [-60^\circ, 60^\circ]$$

$$\text{R} = [-\frac{1}{2}, \frac{1}{2}]$$



$$13. y = 4 \sin \left(2x - \frac{\pi}{2} \right) + 2$$

$$\text{Amp} = 4$$

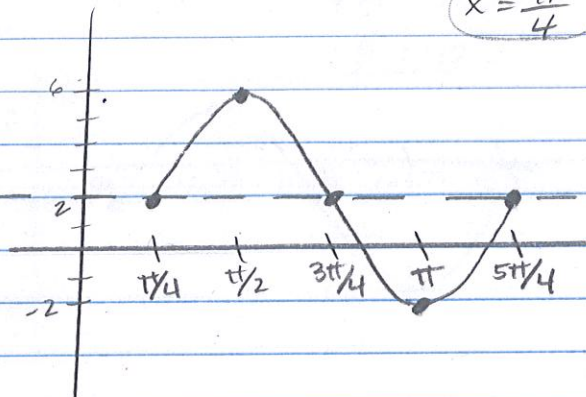
$$\text{Period} = \frac{2\pi}{2} = \pi$$

$$\text{PS} = \frac{\pi}{4}$$

$$\text{VS} = 2$$

$$\text{D} = \left[\frac{\pi}{4}, \frac{5\pi}{4} \right]$$

$$\text{R} = [-2, 6]$$



$$2x - \frac{\pi}{2} = 0 \quad 2x - \frac{\pi}{2} = 2\pi$$

$$2x = \frac{\pi}{2} \quad 2x = \frac{4\pi}{2} + \frac{\pi}{2}$$

$$x = \frac{\pi}{4} \quad (\frac{1}{2}) \quad 2x = \frac{5\pi}{2} \quad (\frac{1}{2})$$

$$x = \frac{5\pi}{4}$$

$$14. y = -5 \sin \left(\frac{x}{2} + \frac{\pi}{2} \right) - 3$$

↳ Reflect

$$\text{Amp} = 5$$

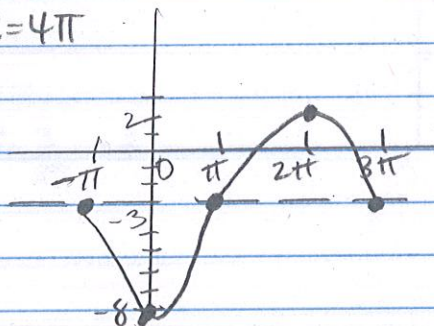
$$\text{Period} = \frac{2\pi}{1/2} = 2\pi \cdot 2 = 4\pi$$

$$\text{PS} = -\pi$$

$$\text{VS} = -3$$

$$\text{D} = [-\pi, 3\pi]$$

$$\text{R} = [-8, 2]$$



$$\frac{x}{2} + \frac{\pi}{2} = 0 \quad \frac{x}{2} + \frac{\pi}{2} = 2\pi$$

$$(2) \frac{x}{2} = -\frac{\pi}{2} \quad (1) \quad \frac{x}{2} = \frac{4\pi}{2} - \frac{\pi}{2}$$

$$x = -\pi$$

$$(2) \frac{x}{2} = \frac{3\pi}{2} \quad (2)$$

$$x = 3\pi$$

$$15. y = 2 \cos(4\theta + 180^\circ) + 1$$

$$\text{Amp} = 2$$

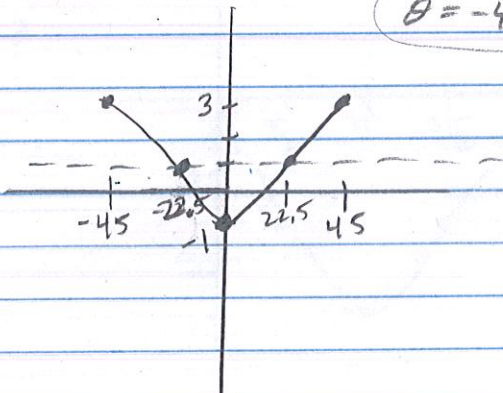
$$\text{Period} = \frac{360}{4} = 90^\circ$$

$$\text{PS} = -45^\circ$$

$$\text{VS} = 1$$

$$\text{D: } [-45, 45]$$

$$\text{R: } [-1, 3]$$



$$4\theta + 180 = 0 \quad 4\theta + 180 = 360$$

$$4\theta = -180 \quad 4\theta = 180$$

$$\theta = -45^\circ \quad \theta = 45^\circ$$

16. $y = 3 \sin(8\theta - 720^\circ) - 1$

$8\theta - 720^\circ = 0$

$8\theta - 720^\circ = 360^\circ$

$8\theta = 720^\circ$

$8\theta = 1080^\circ$

$\theta = 90^\circ$

$\theta = 135^\circ$

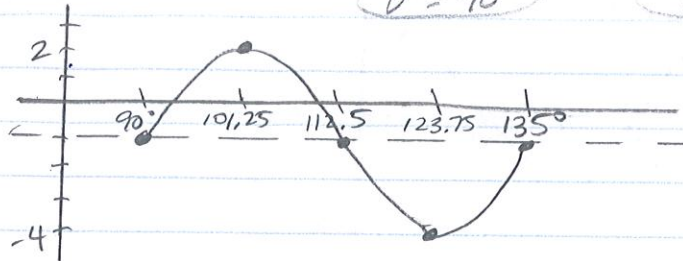
Amp = 3
 $P = \frac{360}{8} = 45^\circ$

PS = 90°

VS = -1

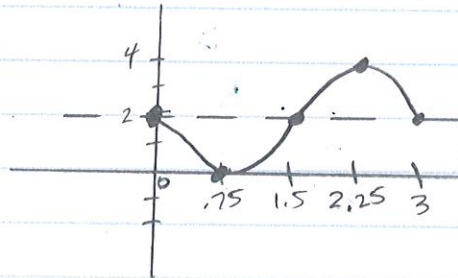
D: $[90^\circ, 135^\circ]$

R: $[-4, 2]$



17. $y = 2 - 2 \sin \frac{2\pi x}{3}$
 $y = -2 \sin \frac{2\pi x}{3} + 2$

Reflection



Amp = 2

Period = $\frac{2\pi}{2\pi/3} = 2\pi \cdot \frac{3}{2\pi} = 3$

PS = None

VS = 2

D = $[0, 3]$

R = $[0, 4]$

18. $y = 2 \cos x - 3$

Amp = 2

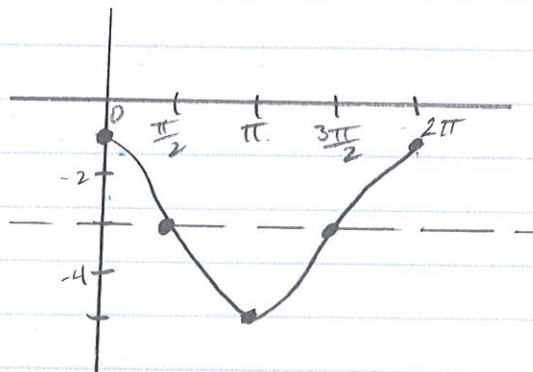
Period = $\frac{2\pi}{1} = 2\pi$

PS = None

VS = -3

D: $[0, 2\pi]$

R: $[-5, -1]$



$$19. y = \frac{2}{3} \cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$$

$$\frac{x}{2} - \frac{\pi}{4} = 0 \quad \frac{x}{2} - \frac{\pi}{4} = 2\pi$$

$$(\circledast) \frac{x}{2} = \frac{\pi}{4} \quad (\circledast) \frac{x}{2} = \frac{8\pi + \pi}{4}$$

$$x = \frac{\pi}{2}$$

$$(\circledast) \frac{x}{2} = \frac{9\pi}{4} \quad (\circledast)$$

$$x = \frac{9\pi}{2}$$

$$\text{Amp} = \frac{2}{3}$$

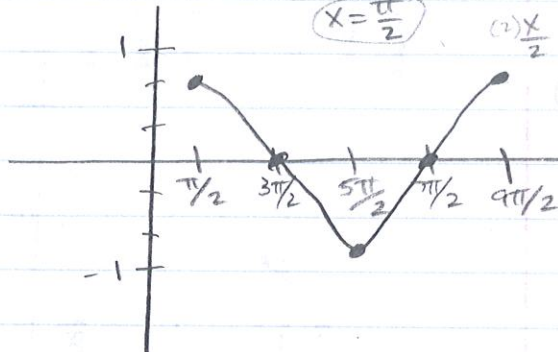
$$\text{Period} = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$$

$$\text{PS} = \left[\frac{\pi}{2}, \frac{9\pi}{2}\right]$$

VS = None

D:

$$\text{R: } \left[-\frac{2}{3}, \frac{2}{3}\right]$$



$$20. y = -3 \cos(6x + \pi) - 2$$

↑ Reflection!

$$6x + \pi = 0$$

$$6x + \pi = 2\pi$$

$$6x = -\pi$$

$$6x = \pi$$

$$x = \frac{-\pi}{6}$$

$$x = \frac{\pi}{6}$$

$$\text{Amp} = |-3| = 3$$

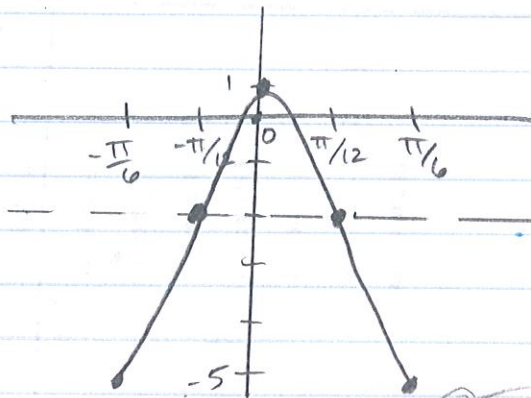
$$\text{period} = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$\text{PS} = -\frac{\pi}{6}$$

$$\text{VS} = -2$$

$$\text{D: } \left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$$

$$\text{R: } [-5, 1]$$



$$\frac{\pi}{6} \cdot \frac{1}{2} = \frac{\pi}{12}$$