

State the amplitude, period, phase shift and vertical shift for each of the following functions. Then graph one complete period of each, remembering to label the tick divisions on both your horizontal axis and vertical axis. Also state the domain and range of one period using interval notation.

\*\*\* Remember:  $\theta \rightarrow$  degrees and  $x \rightarrow$  radians.

1.  $y = \frac{2}{3} \cos(3x + \pi)$

$3x + \pi = 0 \quad 3x + \pi = 2\pi$   
 $3x = -\pi \quad 3x = \pi$   
 $x = -\pi/3 \quad x = \pi/3$

amp =  $\frac{2}{3}$   
 period =  $\frac{2\pi}{3}$   
 PS =  $-\pi/3$   
 VS = none  
 D:  $[-\pi/3, \pi/3]$   
 R:  $[-\frac{2}{3}, \frac{2}{3}]$

2.  $y = -2 \sin(4\theta + 40^\circ)$

amp =  $|-2| = 2$   
 Period =  $\frac{360}{4} = 90^\circ$   
 PS =  $-10^\circ$   
 VS = none  
 D:  $[-10^\circ, 80^\circ]$   
 R:  $[-2, 2]$

$4\theta + 40 = 0 \quad 4\theta + 40 = 360$   
 $4\theta = -40 \quad 4\theta = 320$   
 $\theta = -10^\circ \quad \theta = 80^\circ$

3.  $y = -\cos 3\theta + 2$

amp =  $|-1| = 1$   
 period =  $\frac{360}{3} = 120^\circ$   
 PS = none  
 VS = 2  
 D:  $[0, 120^\circ]$   
 R:  $[1, 3]$

4.  $f(x) = 5 \sin 2x - 3$

amp = 5  
 period =  $\frac{2\pi}{2} = \pi$   
 PS = none  
 VS = -3  
 D:  $[0, \pi]$   
 R:  $[-8, 2]$

5.  $y = \cos\left(3x - \frac{\pi}{2}\right)$

$3x - \frac{\pi}{2} = 0 \quad 3x - \frac{\pi}{2} = 2\pi$   
 $(\frac{1}{3})3x = \frac{\pi}{2}(\frac{1}{3}) \quad 3x = \frac{4\pi + \pi}{2}$   
 $x = \frac{\pi}{6} \quad (\frac{1}{3})3x = \frac{5\pi}{2}(\frac{1}{3})$   
 $x = \frac{5\pi}{6}$

amp = 1  
 period =  $\frac{2\pi}{3}$   
 PS =  $\frac{\pi}{6}$   
 VS = none  
 D:  $[\frac{\pi}{6}, \frac{5\pi}{6}]$   
 R:  $[-1, 1]$

6.  $y = 3 \sin(-2x) + 2 \Rightarrow y = -3 \sin(2x) + 2$

amp = 3  
 period =  $\frac{2\pi}{2} = \pi$   
 PS = none  
 VS = 2  
 D:  $[0, \pi]$   
 R:  $[-1, 5]$

7.  $y = 2 \sin\left(\frac{1}{2}x + \frac{\pi}{2}\right) + 4$

$\frac{1}{2}x + \frac{\pi}{2} = 0 \quad \frac{1}{2}x + \frac{\pi}{2} = 2\pi$   
 $(\frac{1}{2})x = -\frac{\pi}{2}(\frac{1}{2}) \quad \frac{1}{2}x = \frac{4\pi + \pi}{2}$   
 $x = -\pi \quad \frac{1}{2}x = \frac{5\pi}{2}$   
 $x = 3\pi$

amp = 2  
 period =  $\frac{2\pi}{1/2} = 2\pi \cdot 2 = 4\pi$   
 PS =  $-\pi$   
 VS = 4  
 D:  $[-\pi, 3\pi]$   
 R:  $[2, 6]$

8.  $y = -2 \cos \frac{3\pi x}{4} + 1$

amp =  $|-2| = 2$   
 Period =  $\frac{2\pi}{3\pi/4} = 2\pi \cdot \frac{4}{3\pi} = \frac{8}{3}$   
 PS = none  
 VS = 1  
 D:  $[0, 8/3]$   
 R:  $[-1, 3]$

9. Complete the following statements:  $4 \sin(-3x) = \underline{-4(3x)}$  (b)  $4 \cos(-3x) = \underline{4 \cos(3x)}$

10. A negative phase shift will move the graph of a sinusoid to the: (a) right (b) left

A positive phase shift will move the graph of a sinusoid to the: (a) right (b) left

11. The horizontal axis represents the \_\_\_\_\_ of a sinusoidal function. (a) phase shift (b) vertical shift

12. Given: The domain of a sinusoid is  $[-82^\circ, 998^\circ]$

$$998 - (-82) = 1080^\circ$$

(a) The phase shift of the function is  $\underline{-82^\circ}$  (b) The period of the function is  $\underline{1080^\circ}$

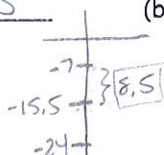
(c) List the 5 ticks that will appear along the x axis when this function is graphed.

$$\frac{998 - (-82)}{2} = 458^\circ$$

$$\boxed{-82^\circ, 188^\circ, 458^\circ, 728^\circ, 998^\circ}$$

13. Given: The range of a sinusoidal function is  $[-24, -7]$ . Find the following:

(a) amplitude of the function  $\underline{8.5}$  (b) vertical shift of the function  $\underline{-15.5}$



$$\frac{-24 + (-7)}{2} = \boxed{-15.5}$$

14. Given:  $y = -4 \cos(5x/3 - 4\pi/3) + 1$  Find the following:

Amplitude  $\underline{4}$  Period  $\underline{\frac{6\pi}{5}}$  Phase shift  $\underline{\frac{4\pi}{5}}$  Vertical shift  $\underline{1}$

$$\hookrightarrow \frac{2\pi}{5/3} = 2\pi \cdot \frac{3}{5} = \frac{6\pi}{5}$$

$$\hookrightarrow \frac{5x}{3} - \frac{4\pi}{3} = 0$$

$$\left(\frac{3}{5}\right) \frac{5x}{3} = \frac{4\pi}{3} \left(\frac{3}{5}\right)$$

$$x = \frac{4\pi}{5}$$

15. Write the equation of a sine function whose domain is  $[8^\circ, 68^\circ]$  and whose range is  $[4, 7]$ . Period =  $60^\circ$

$$\boxed{y = \pm 1.5 \sin(6\theta - 48^\circ) + 5.5}$$

$$\frac{360}{b} = 60^\circ$$

$$\boxed{b = 6^\circ}$$

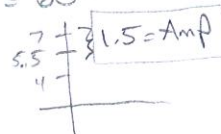
$$bx - c = 0$$

$$6(8) - c = 0$$

$$48 - c = 0$$

$$\boxed{48 = c}$$

$$\frac{7+4}{2} = \boxed{5.5 = VS}$$



16. If you are writing the equation of a sinusoidal function and you are given only the maximum and/or minimum values of the function, then the BEST function to choose would be: (a) sine (b) cosine