

What does it mean to say $\sqrt{4}$? 2 is the number that can be squared to get 4

So, in general \sqrt{x} means: What number could be squared to get x?

Square Root is the INVERSE of Squared.

Memorize the following square roots. These are the most common!

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|-----------------|-------------------|-------------------|-------------------|
| $\sqrt{1} = 1$ | $\sqrt{4} = 2$ | $\sqrt{9} = 3$ | $\sqrt{16} = 4$ |
| $\sqrt{25} = 5$ | $\sqrt{36} = 6$ | $\sqrt{49} = 7$ | $\sqrt{64} = 8$ |
| $\sqrt{81} = 9$ | $\sqrt{100} = 10$ | $\sqrt{121} = 11$ | $\sqrt{144} = 12$ |
- $\sqrt{12 \cdot 12} = 12$

Simplify each radical expression: Rewrite each radical as a product of two factors (one being the largest perfect square each number is divisible by). 1 4 9 16 25 36 49 64 81 100

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|--|---|--|
| 1. $\sqrt{12}$
$\sqrt{4} \sqrt{3}$
$2\sqrt{3}$ | 2. $\sqrt{28}$
$\sqrt{4} \sqrt{7}$
$2\sqrt{7}$ | 3. $\sqrt{35}$
$\sqrt{5} \sqrt{7}$
Prime |
| 4. $\sqrt{20}$
$\sqrt{4} \sqrt{5}$
$2\sqrt{5}$ | 5. $\sqrt{72}$
$\sqrt{36} \sqrt{2}$
$6\sqrt{2}$ | 6. $\sqrt{128}$
$\sqrt{64} \sqrt{2}$
$8\sqrt{2}$ |

When simplifying radicals with a coefficient, remember: "outsides with outsides" and "insides with insides."

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|--|--|--|
| 1. $3\sqrt{8}$
$3\sqrt{4} \sqrt{2}$
$3 \cdot 2\sqrt{2}$
$6\sqrt{2}$ | 2. $10\sqrt{36}$
$10 \cdot 6$
60 | 3. $6\sqrt{12}$
$6\sqrt{4} \sqrt{3}$
$6 \cdot 2\sqrt{3}$
$12\sqrt{3}$ |
| 4. $-20\sqrt{125}$
$-20\sqrt{25} \sqrt{5}$
$-20 \cdot 5\sqrt{5}$
$-100\sqrt{5}$ | 5. $6\sqrt{45}$
$6\sqrt{9} \sqrt{5}$
$6 \cdot 3\sqrt{5}$
$18\sqrt{5}$ | 6. $-\sqrt{49}$
-7 |

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Dividing Radicals. (Remember to simplify completely.)

1. $\frac{\sqrt{30}}{\sqrt{15}} = \sqrt{\frac{30}{15}} = \boxed{\sqrt{2}}$

2. $\frac{\sqrt{21}}{\sqrt{7}} = \boxed{\sqrt{3}}$

3. $\frac{\sqrt{40}}{\sqrt{8}} = \boxed{\sqrt{5}}$

4. $\frac{6\sqrt{200}}{\sqrt{20}} = \boxed{6\sqrt{10}}$

5. $\frac{8\sqrt{70}}{2\sqrt{10}} = \boxed{4\sqrt{7}}$

6. $\frac{\sqrt{16}}{\sqrt{2}} = \frac{\sqrt{8}}{\sqrt{4}} = \frac{\sqrt{4} \sqrt{2}}{2} = \boxed{\sqrt{2}}$

IMPORTANT: * When simplifying, never leave a radical in the denominator of a fraction. *

Always rationalize the denominator.

When the denominator is a monomial, multiply both the numerator and the denominator by whatever makes the denominator an expression that can be simplified so that it is no longer a radical.

Example 1: Simplify $\frac{2}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$

Example 2: Simplify $\frac{10}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{10\sqrt{6}}{6} = \frac{5\sqrt{6}}{3}$

$\sqrt{7} \cdot \sqrt{7} = \sqrt{49} = 7$

1 4 9 16 25 36 49

Sometimes you need to multiply by whatever makes the denominator a perfect square.

Example 3: Simplify $\frac{5}{\sqrt{8}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{16}} = \frac{5\sqrt{2}}{4}$

Example 4: Simplify $\frac{2}{\sqrt{18}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{36}} = \frac{2\sqrt{2}}{6} = \frac{\sqrt{2}}{3}$

If the radicand is a fraction, rewrite the numerator and denominator as two separate radicals.

Example 5: Simplify $\sqrt{\frac{1}{5}} = \frac{\sqrt{1}}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

Example 6: Simplify $\sqrt{\frac{9}{11}} = \frac{3}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} = \frac{3\sqrt{11}}{11}$

Example 7: Simplify $\frac{\sqrt{45}}{\sqrt{5}} \sqrt{9} = \boxed{3}$

Example 8: Simplify $\frac{\sqrt{12}}{\sqrt{24}} = \sqrt{\frac{12}{24}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

Example 9: Simplify $\frac{\sqrt{21}}{\sqrt{18}} = \frac{\sqrt{7}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$

$$\boxed{\frac{\sqrt{42}}{6}}$$

Example 10: Simplify $\frac{12}{5\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{12\sqrt{6}}{5 \cdot 6} = \frac{12\sqrt{6}}{30}$

$$= \boxed{\frac{2\sqrt{6}}{5}}$$

Dividing Radicals Practice

1. $\frac{8}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{8\sqrt{5}}{5}}$

2. $\frac{10}{3\sqrt{20}} = \frac{10}{6\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$$\begin{aligned} 3\sqrt{20} &= 3\sqrt{4 \cdot 5} \\ &= 3 \cdot 2 \cdot \sqrt{5} \\ &= 6\sqrt{5} \end{aligned}$$

$$= \frac{10\sqrt{5}}{30}$$

$$= \boxed{\frac{\sqrt{5}}{3}}$$

3. $\frac{\sqrt{90}}{\sqrt{5}} \sqrt{18}$

$$\frac{\sqrt{9} \sqrt{2}}{3\sqrt{2}} = \boxed{3\sqrt{2}}$$

4. $\sqrt{\frac{4}{3}} = \frac{\sqrt{4}}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

5. $\frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = \boxed{3\sqrt{2}}$

6. $\sqrt{\frac{125}{5}} = \sqrt{25} = \boxed{5}$

7. $\frac{3\sqrt{6}}{2\sqrt{3}} = \boxed{\frac{3\sqrt{2}}{2}}$

$$\sqrt{\cancel{x} \cancel{x} \cancel{x} \cancel{x} \cancel{x}}$$

Exponent Even

$$\sqrt{x^6} = x^3$$

Simplify each radical expression with variables.

1. $\sqrt{a^8 b^{11} c}$

$$a^4 b^5 \sqrt{bc}$$

2. $\sqrt{30 a^2 b^3}$

$$ab \sqrt{30b}$$

Exponent Odd

$$\sqrt{x^7} = \sqrt{x^6} \sqrt{x} = x^3 \sqrt{x}$$

3. $\sqrt{75 m^3 n^5}$

$$\sqrt{25} \cdot \sqrt{3}$$

$$5 m n^2 \sqrt{3 m n}$$

4. $\sqrt{80 x^{100} y^{49}}$

$$\sqrt{16} \sqrt{5}$$

$$4 x^{50} y^{24} \sqrt{5y}$$

5. $\sqrt{20 x y^{21}}$

$$\sqrt{4} \cdot \sqrt{5}$$

$$2 y^{10} \sqrt{5xy}$$

6. $-4 \sqrt{50 x^4 y^{98}}$

$$\sqrt{25} \sqrt{2}$$

$$-4 \cdot 5 x^2 y^{49} \sqrt{2}$$

$$-20 x^2 y^{49} \sqrt{2}$$

Multiplying Radicals. (Remember to simplify completely.) "outsides with outsides" and "insides with insides."

1. $\sqrt{7} \cdot \sqrt{3}$

$$\sqrt{21}$$

2. $2\sqrt{6} \cdot 3\sqrt{11}$

$$6\sqrt{66}$$

3. $\sqrt{10} \cdot \sqrt{5}$

$$\sqrt{50}$$

$$\sqrt{25} \cdot \sqrt{2}$$

$$5\sqrt{2}$$

4. $2\sqrt{8} \cdot \sqrt{3}$

$$2\sqrt{24}$$

$$2\sqrt{4} \sqrt{6}$$

$$2 \cdot 2\sqrt{6}$$

$$4\sqrt{6}$$

7. $\sqrt{a^2 b} \cdot \sqrt{ab^2}$

$$\sqrt{a^3 b^3}$$

$$ab\sqrt{ab}$$

5. $-5\sqrt{12} (3\sqrt{6})$

$$-15\sqrt{72}$$

$$-15\sqrt{36} \sqrt{2}$$

$$-15 \cdot 6\sqrt{2}$$

$$-90\sqrt{2}$$

8. $\sqrt{2a^3} \cdot \sqrt{8a^5 b^2}$

$$\sqrt{16a^8 b^2}$$

$$4a^4 b$$

6. $-\sqrt{18} (-9\sqrt{2})$

$$9\sqrt{36}$$

$$9 \cdot 6$$

$$54$$

9. $\sqrt{5x} (\sqrt{2x})$

* multiplying with variables, Add exponents!

10. $\sqrt{12x^7} \cdot \sqrt{21x}$

$$\sqrt{252x^8}$$

$$\sqrt{36} \sqrt{7}$$

$$6x^4 \sqrt{7}$$

11. $2\sqrt{27x} \cdot 6\sqrt{3x^7}$

$$12\sqrt{81x^8}$$

$$12 \cdot 9x^4$$

$$108x^4$$

12. $\sqrt{8a^{10} b^3} \cdot \sqrt{48a^3 b}$

$$\sqrt{384 a^{13} b^4}$$

$$\sqrt{64} \sqrt{6}$$

$$8a^6 b^2 \sqrt{6a}$$