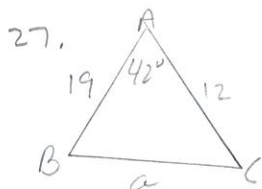


$$a^2 = b^2 + c^2 - 2bc \cos A$$

Law of Sines and Law of Cosines



$$\begin{aligned} A &= 42^\circ & a &= 12.9 \\ B &= 39^\circ & b &= 12 \\ C &= 99^\circ & c &= 19 \end{aligned}$$

$$a^2 = 12^2 + 19^2 - 2(12)(19)\cos 42^\circ$$

$$a^2 = 505 - 338.87$$

$$a^2 = 166.13$$

$$\boxed{a = 12.9}$$

$$180 - 42 - 39 = 99$$

$$\boxed{C = 99^\circ}$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

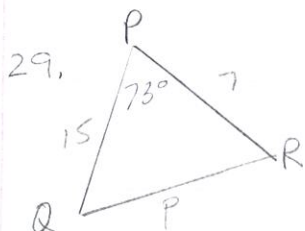
$$12^2 = \underbrace{12.9^2 + 19^2}_{527.41} - 2(12.9)(19)\cos B$$

$$144 = 527.41 - 490.2 \cos B$$

$$-383.41 = -490.2 \cos B$$

$$\frac{383.41}{490.2} = \cos B$$

$$B = \cos^{-1} \frac{383.41}{490.2} \quad \boxed{B = 39^\circ}$$



$$\begin{aligned} P &= 73^\circ \\ Q &= 27^\circ \\ R &= 80^\circ \end{aligned}$$

$$\begin{aligned} p &= 14.6 \\ q &= 7 \\ r &= 15 \end{aligned}$$

$$p^2 = r^2 + q^2 - 2(r)(q)\cos P$$

$$p^2 = 7^2 + 15^2 - 2(7)(15)\cos 73^\circ$$

$$p^2 = 212.6$$

$$\boxed{p = 14.6}$$

$$180 - 73 - 27 = 80$$

$$\boxed{m\angle R = 80^\circ}$$

$$7^2 = \underbrace{15^2 + 14.6^2}_{438.16} - 2(15)(14.6)\cos Q$$

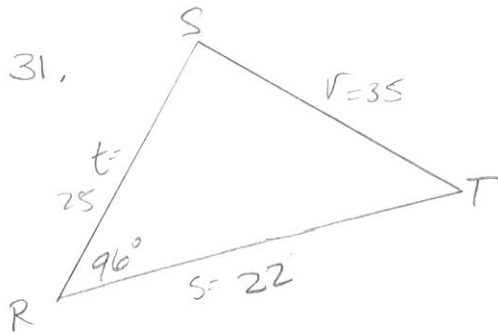
$$49 = 438.16 - 438 \cos Q$$

$$-389.16 = -438 \cos Q$$

$$Q = \cos^{-1} \frac{389.16}{438}$$

$$\boxed{Q = 27^\circ}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$



$$\begin{array}{ll} R = 96^\circ & r = 35 \\ S = 39^\circ & s = 22 \\ T = 45^\circ & t = 25 \end{array}$$

$$180 - 96 - 45 = 39^\circ$$

$$\boxed{S = 39^\circ}$$

$$35^2 = 25^2 + 22^2 - 2(25)(22) \cos R$$

$$1225 = 1109 - 1100 \cos R$$

$$116 = -1100 \cos R$$

$$\frac{116}{-1100} = \cos R$$

$$R = \cos^{-1}\left(-\frac{116}{1100}\right) \quad \boxed{R = 96^\circ}$$

$$25^2 = 35^2 + 22^2 - 2(35)(22) \cos T$$

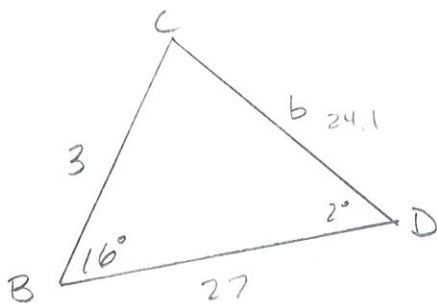
$$625 = 1709 - 1540 \cos T$$

$$-1084 = -1540 \cos T$$

$$\frac{-1084}{-1540} = \cos T$$

$$T = \cos^{-1}\left(\frac{1084}{1540}\right) \quad \boxed{T = 45^\circ}$$

33.



$$\begin{array}{ll} B = 16^\circ & b = 24.1 \\ C = 162^\circ & c = 27 \\ D = 2^\circ & d = 3 \end{array}$$

$$180 - 16 - 2 = 162^\circ$$

$$\boxed{C = 162^\circ}$$

$$b^2 = 27^2 + 3^2 - 2(27)(3) \cos 16^\circ$$

$$b^2 = 738 - 162 \cos 16^\circ$$

$$b^2 = 582.2756$$

$$\boxed{b = 24.1}$$

$$3^2 = 24.1^2 + 27^2 - 2(24.1)(27) \cos D$$

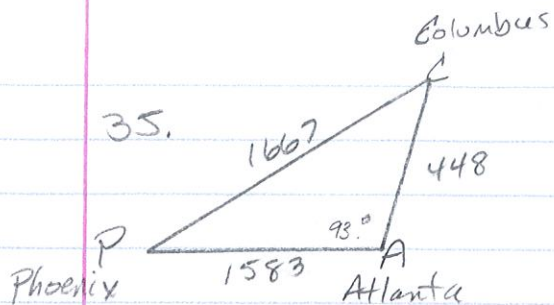
$$9 = 1309.81 - 1301.4 \cos D$$

$$-1300.81 = -1301.4 \cos D$$

$$\frac{-1300.81}{-1301.4} = \cos D$$

$$D = \cos^{-1}\left(\frac{1300.81}{1301.4}\right)$$

$$\boxed{D = 2^\circ}$$



$$1667^2 = 1583^2 + 448^2 - 2(1583)(448)\cos A$$

$$72296 = -1418368 \cos A$$

$$A = \cos^{-1}\left(\frac{-72296}{1418368}\right)$$

$$\boxed{A = 93^\circ}$$

$$A = 93^\circ \quad a = 1667$$

$$C = 71^\circ \quad c = 1583$$

$$P = 16^\circ \quad p = 448$$

$$448^2 = 1583^2 + 1667^2 - 2(1583)(1667)\cos P$$

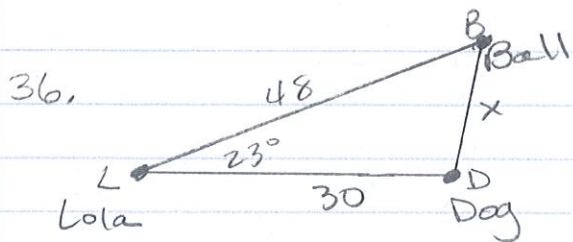
$$-5084074 = -5277722 \cos P$$

$$P = \cos^{-1}\left(\frac{5084074}{5277722}\right)$$

$$\boxed{P = 16^\circ}$$

$$180 - 93 - 16 = 71$$

$$\boxed{C = 71^\circ}$$



$$x^2 = 48^2 + 30^2 - 2(48)(30)\cos 23^\circ$$

$$x^2 = 552.946$$

$$\boxed{x = 23.5 \text{ ft.}}$$

## 4-7 The Law of Sines and the Law of Cosines

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

27.  $\triangle ABC$ , if  $A = 42^\circ$ ,  $b = 12$ , and  $c = 19$

29.  $\triangle PQR$ , if  $P = 73^\circ$ ,  $q = 7$ , and  $r = 15$

31.  $\triangle RST$ , if  $r = 35$ ,  $s = 22$ , and  $t = 25$

33.  $\triangle BCD$ , if  $B = 16^\circ$ ,  $c = 27$ , and  $d = 3$

35. **AIRPLANES** During her shift, a pilot flies from the Columbus to Atlanta, a distance of 448 miles, and then on to the Phoenix, a distance of 1583 miles. From Phoenix, she returns home to Columbus, a distance of 1667 miles. Determine the angles of the triangle created by her flight path.

36. **CATCH** Lola rolls a ball on the ground at an angle of  $23^\circ$  to the right of her dog Buttons. If the ball rolls a total distance of 48 feet, and she is standing 30 feet away, how far will Buttons have to run to retrieve the ball?