

Algebraic Properties and Proofs

Name _____

You have solved algebraic equations for a couple years now, but now it is time to justify the steps you have practiced and now take without thinking... and acting without thinking is a dangerous habit!

The following is a list of the reasons one can give for each algebraic step one may take.

KNOW!

ALGEBRAIC PROPERTIES OF EQUALITY	
ADDITION PROPERTY OF EQUALITY	If $a = b$, then $a + c = b + c$
SUBTRACTION PROPERTY OF EQUALITY	If $a = b$, then $a - c = b - c$
MULTIPLICATION PROPERTY OF EQUALITY	If $a = b$, then $a \cdot c = b \cdot c$
DIVISION PROPERTY OF EQUALITY	If $a = b$, then $\frac{a}{c} = \frac{b}{c}$
DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION or OVER SUBTRACTION	$a(b + c) = ab + ac$ $a(b - c) = ab - ac$
SUBSTITUTION PROPERTY OF EQUALITY	If $a = b$, then b can be substituted for a in any equation or expression
REFLEXIVE PROPERTY OF EQUALITY	For any real number a , $a = a$
SYMMETRIC PROPERTY OF EQUALITY	If $a = b$, then $b = a$
TRANSITIVE PROPERTY OF EQUALITY	If $a = b$ and $b = c$, then $a = c$

Complete the following algebraic proofs using the reasons above. If a step requires simplification by combining like terms, write *simplify*.

2 column Proof - It's a way to organize your thinking!
- Always start with the given statement.

Given: $3x + 12 = 8x - 18$

Prove: $x = 6$

Statements	Reasons
1. $3x + 12 = 8x - 18$	1. Given
2. $12 = 5x - 18$	2. Subtraction Property
3. $30 = 5x$	3. Addition Property
4. $6 = x$	4. Division Property
5. $x = 6$	5. Symmetric Property

Given: $3k + 5 = 17$

Prove: $k = 4$

	Statements	Reasons
1.	$3k + 5 = 17$	1. Given
2.	$3k = 12$	2. Subtraction Property
3.	$k = 4$	3. Division Property

Given: $-6a - 5 = -95$

Prove: $a = 15$

	Statements	Reasons
	$-6a - 5 = -95$	Given
	$-6a = -90$	Addition Property
	$a = 15$	Division Property

Given: $3(5x + 1) = 13x + 5$

Prove: $x = 1$

	Statements	Reasons
	$3(5x + 1) = 13x + 5$	Given
	$15x + 3 = 13x + 5$	Distributive Property
	$2x + 3 = 5$	Subtraction Property
	$2x = 2$	Subtraction Property
	$x = 1$	Division Property

HOMEWORK

WORKSHEET: Algebraic Proof

Name: _____

Solve each equation. Write a reason for every step.

Statement	Reasons
1. $4x = 12x + 32$	
$4x = 12x + 32$	Given
$-12x \quad -12x$	
$-8x = 32$	Subtraction P.
$x = -4$	Division P.

Statement	Reasons
2. $28 + 12x = 8x - 4$	
$28 + 12x = 8x - 4$	Given
$-8x \quad -8x$	
$28 + 4x = -4$	Subtraction P.
$-28 \quad -28$	
$4x = -32$	Subtraction P.
$x = -8$	Division P.

Statement	Reasons
3. $60x + 153 = 9x + 51$	
$60x + 153 = 9x + 51$	Given
$-9x \quad -9x$	
$51x + 153 = 51$	Subtraction P.
$-153 \quad -153$	
$51x = -102$	Subtraction P.
$x = -2$	Division P.

Statement	Reasons
4. $-4x + 10 = -5x + 18$	
$-4x + 10 = -5x + 18$	Given
$x + 10 = 18$	Addition P.
$x = 8$	Subtraction P.

Statement	Reasons
5. $-3(x + 2) = 16 - x$	
$-3(x + 2) = 16 - x$	Given
$-3x - 6 = 16 - x$	Distributive P.
$+3x \quad +3x$	
$-6 = 16 + 2x$	Addition P.
$-16 \quad -16$	
$-22 = 2x$	Subtraction P.
$-11 = x$	Division P.
$x = -11$	Symmetric P.

Statement	Reasons
6. $-x - 2(9 - 8x) = 12$	
$-x - 2(9 - 8x) = 12$	Given
$-x - 18 + 16x = 12$	Distributive P.
$-18 + 15x = 12$	Combining like terms
$15x = 30$	Addition P.
$x = 2$	Division P.

Statement	Reasons
7. $6(x - 6) = x(16 - 7)$	
$6(x - 6) = x(16 - 7)$	Given
$6x - 36 = 9x$	Distributive P.
$-6x \quad -6x$	
$-36 = 3x$	Subtraction P.
$-12 = x$	Division P.
$x = -12$	Symmetric P.

Statement	Reasons
8. $\frac{1}{4}x + 10 = 2$	
$\frac{1}{4}x + 10 = 2$	Given
$-10 \quad -10$	
$(4) \frac{1}{4}x = -8(4)$	Subtraction P.
$x = -32$	Multiplication P.

Worksheet: Algebraic Proofs

Complete the proofs below.

1. **Given:** $3x + 12 = 8x - 18$
Prove: $x = 6$

Statements	Reasons
$3x + 12 = 8x - 18$	Given
$12 = 5x - 18$	Subtraction Property
$30 = 5x$	Addition Property
$6 = x$	Division Property
$x = 6$	Symmetric Property

2. **Given:** $3k + 5 = 17$
Prove: $k = 4$

Statements	Reasons
$3k + 5 = 17$	Given
$3k = 12$	Subtraction P
$k = 4$	Division P

3. **Given:** $-6a - 5 = -95$
Prove: $a = 15$

Statements	Reasons
$-6a - 5 = -95$	Given
$-6a = -90$	Addition P
$a = 15$	Division P

Complete the proofs.

4. **Given:** $3(5x+1) = 13x+5$

Prove: $x=1$

Statements	Reasons
$3(5x+1) = 13x+5$	Given
$15x+3 = 13x+5$	Distributive P.
$2x+3 = 5$	Subtraction P.
$2x = 2$	Subtraction P.
$x = 1$	Division P.

5. **Given:** $7y - 84 = 2y + 61$

Prove: $y = 29$

Statements	Reasons
$7y - 84 = 2y + 61$	Given
$5y - 84 = 61$	Subtraction P.
$5y = 145$	Addition P.
$y = 29$	Division P.

6. **Given:** $\frac{3}{5}x = -9$

Prove: $x = -15$

Statements	Reasons
$\left(\frac{5}{3}\right) \frac{3}{5}x = -9\left(\frac{5}{3}\right)$	Given
$x = -15$	Multiplication Property

Write the letter of each property next to its definition. The letters a , b , and c represent real numbers. You may use a property more than once.

F 10. If $5 = x$, then $x = 5$

C 11. If $a = b$, then $2a = 2b$

F 12. $m = n$, so $n = m$

E 13. $7 = 7$

A 14. If $a = b$, then $a + 8 = b + 8$

I 15. $9(x + y) = 9x + 9y$

G 16. If $x = 3$ and $3 = y$, then $x = y$

G 17. $p = q$ and $q = -1$, so $p = -1$

D 18. If $3x = 27$, then $x = 9$

D 19. If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$

H 20. If $a = b$, then b can be replaced by a in an expression.

B 21. If $a = b$, then $a - c = b - c$

A. Addition Property of Equality

B. Subtraction Property of Equality

C. Multiplication Property of Equality

D. Division Property of Equality

E. Reflexive Property of Equality

F. Symmetric Property of Equality

G. Transitive Property of Equality

H. Substitution Property

I. Distributive Property