Multiplying Matrices

Properties of Matrix Operations

Associative Property of Addition	$(A + B) + C = A + (B + C)$
Commutative Property of Addition	$A + B = B + A$
Distributive Property (with a scalar)	$c(A \pm B) = cA \pm cB$

c real number

Properties of Matrix Multiplication

Associative Property of Matrix Multiplication	$A(BC) = (AB)C$
Left Distributive Property	$A(B + C) = AB + AC$
Right Distributive Property	$(A + B)C = AC + BC$
Associative Property of Scalar Multiplication	$c(AB) = (cA)B = A(cB)$

4. $A = \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$. Find B(A + C) and BA + BC.

B(A+C)

$$\begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}\left(\begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}+\begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}\right)$$

$$\begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 0(2)+1(3) & 0(1)+1(3) \\ -3(2)+^-2(3) & -3(1)+^-2(3) \end{bmatrix}$$

$$\begin{bmatrix} 3 & 3 \\ -12 & -9 \end{bmatrix}$$

BA+BC

$$\begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}+\begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0(2)+1(1) & 0(-2)+1(4) \\ -3(2)+^-2(1) & -3(-2)+^-2(4) \end{bmatrix}+\begin{bmatrix} 0(0)+1(2) & 0(3)+1(-1) \\ -3(0)+^-2(2) & -3(3)+^-2(-1) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 \\ -8 & -2 \end{bmatrix}+\begin{bmatrix} 2 & -1 \\ -4 & -7 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 3 \\ -12 & -9 \end{bmatrix}$$

Multiplying Matrices

A · B = AB

A x B → can the matrices be multiplied?

1. State whether AB is defined. If so, give the dimensions.

   a. A: 2 x 4 (R C) B: 4 x 3 (R C)
   2 x 3 — yes

   b. A: 1 x 4 B: 1 x 4
   No! Middle numbers are not the same.

* middle terms must be the same

2. Find AB if $A = \begin{bmatrix} -1 & 5 \\ 5 & 2 \\ 0 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ 6 & 8 \end{bmatrix}$.

R x C → ↕

3 x 2 (same) 2 x 2 → 3 x 2

$$\begin{bmatrix} -1(4)+5(6) & -1(-3)+5(8) \\ 5(4)+2(6) & 5(-3)+2(8) \\ 0(4)+-4(6) & 0(-3)+-4(8) \end{bmatrix}$$

$$\begin{bmatrix} 26 & 43 \\ 32 & 1 \\ -24 & -32 \end{bmatrix}$$

3. $A = \begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -3 \\ 1 & 2 \end{bmatrix}$.

Find AB 2 x 2 (same) 2 x 2 ✓

$$\begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} -4 & -3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4(-4)+1(1) & 4(-3)+1(2) \\ 0(-4)+-2(1) & 0(-3)+-2(2) \end{bmatrix} = \begin{bmatrix} -15 & -10 \\ -2 & -4 \end{bmatrix}$$

Find BA 2 x 2 (same) 2 x 2 ✓

$$\begin{bmatrix} -4 & -3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -4(4)+-3(0) & -4(1)+-3(-2) \\ 1(4)+2(0) & 1(1)+2(-2) \end{bmatrix} = \begin{bmatrix} -16 & 2 \\ 4 & -3 \end{bmatrix}$$

* Is matrix multiplication commutative? No

Commutative property of multiplication

2 · 3 = 6
3 · 2 = 6