DOT PRODUCT

of VECTORS

What is dot product?

- Dot product (or scalar product) of two vectors is a scalar quantity.
- It is an operation that takes two vectors as input, and returns a scalar number as output.
- The dot product is a value expressing the angular relationship between two vectors

How do I calculate dot product?

if $\vec{v} = \langle v_1, v_2 \rangle$ and $\vec{w} = \langle w_1, w_2 \rangle$,

then the dot product is

$$\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2$$

Examples:

- Find each dot product.

a) $\langle 3,5 \rangle \cdot \langle 2,6 \rangle$ $3 \cdot 2 + 5 \cdot 6 = 6 + 30 = \boxed{36}$

b) $\langle -2,4 \rangle \cdot \langle -1,3 \rangle$

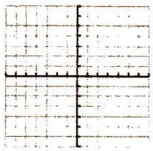
$-2 \cdot -1 + 4 \cdot 3$

$2 + -12$

$\boxed{-10}$

Orthogonal Vectors (Perpendicular)

- Two vectors are said to be orthogonal if $\vec{v} \cdot \vec{w} = 0$. Dot Product = 0
- "Orthogonal" means the vectors are perpendicular (form a 90° angle). ie right angles
- Are $\langle 2, -3 \rangle$ and $\langle 6,4 \rangle$ orthogonal?



$2 \cdot 6 + -3 \cdot 4$
$= 12 + -12$
$= \boxed{0}$ yes

Angle Between Two Vectors

- If θ is between two non-zero vectors $\vec{v}$ and $\vec{w}$, then

$$\cos\theta = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\|\|\vec{w}\|}$$ ← Dot Product

magnitudes: $\sqrt{a^2+b^2}$

Examples: Use [0,360°)

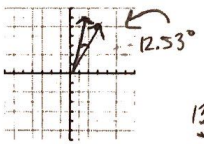
← include zero
← do not include 360°

- Find the angle between …

a) $\langle 3,5 \rangle$ and $\langle 2,6 \rangle$

$\cos\theta = \frac{6+30}{\sqrt{9+25}\sqrt{4+36}}$

$\theta = \cos^{-1}\left(\frac{36}{\sqrt{34}\sqrt{40}}\right)$



$\theta = 12.53°$

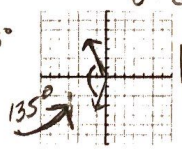
b) $\langle -2,4 \rangle$ and $\langle -1,-3 \rangle$

$\cos\theta = \frac{2-12}{\sqrt{4+16}\sqrt{1+9}}$

$\cos\theta = \frac{-10}{\sqrt{20}\sqrt{10}}$

$\theta = \cos^{-1}\frac{-10}{\sqrt{200}}$

$\theta = 135°$



Backwards Example

- If $\|\vec{a}\| = 4$ and $\|\vec{b}\| = 7$, and $\theta = 112^o$, find $\vec{a} \cdot \vec{b}$.

$\cos\theta = \frac{a \cdot b}{\|a\| \cdot \|b\|}$

$\cos 112° = \frac{x}{4 \cdot 7}$

$\cos 112° = \frac{x}{28}$

$28 \cos 112° = x$

$\boxed{x = -10.49}$