DOT PRODUCT WORKSHEET NAME Key

For the following problems find the dot product of the two vectors:

1. $\vec{a} = \langle 3, 5 \rangle$ and $\vec{b} = \langle -2, 3 \rangle$

$3 \cdot -2 + 5 \cdot 3$

$-6 + 15$ **9**

2. $\vec{c} = \langle 1, -7 \rangle$ and $\vec{d} = \langle -2, -4 \rangle$

$1 \cdot -2 + -7 \cdot -4$

$-2 + 28$

26

3. $\vec{e} = \frac{2}{3}\vec{i} + \frac{3}{2}\vec{i}$ and $\vec{f} = -\frac{5}{2}\vec{i} + \frac{4}{3}\vec{j}$

$\frac{2}{3} \cdot \frac{-5}{2} + \frac{3}{2} \cdot \frac{4}{3} = -\frac{5}{3} + 2 = -\frac{5}{3} + \frac{6}{3} = \boxed{\frac{1}{3}}$

4. $\vec{g} = -3\vec{i} + 5\vec{j}$ and $\vec{h} = -5\vec{i} - 3\vec{j}$

$-3 \cdot -5 + 5 \cdot -3$

$15 - 15$

0

For the following problems find the angle between the two given vectors. Use [0°, 360°). (round to the nearest 100th):

5. $\vec{u} = 3\vec{i} - 5\vec{j}$ and $\vec{v} = -6\vec{i} - 2\vec{j}$ $\left(\cos\theta = \frac{u \cdot v}{\|u\| \cdot \|v\|}\right)$

$\cos\theta = \frac{-18+10}{\sqrt{9+25}\sqrt{36+4}}$

$\cos\theta = \frac{-8}{\sqrt{34}\sqrt{40}}$

$\theta = \cos^{-1}\frac{-8}{\sqrt{1360}}$ $\theta = 102.53°$

6. $\vec{v} = \langle -8, -3 \rangle$ and $\vec{w} = \langle 3, -8 \rangle$

$\cos\theta = \frac{-24+24}{\sqrt{64+9}\sqrt{9+64}}$

$\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\theta = 90°$

7. $\vec{u} = \vec{i} + 3\vec{j}$ and $\vec{v} = -2\vec{j}$

$\langle 1, 3 \rangle$ $\langle 0, -2 \rangle$

$\cos\theta = \frac{0-6}{\sqrt{1+9}\sqrt{0+4}}$

$\cos\theta = \frac{-6}{\sqrt{10}\sqrt{4}}$

$\theta = \cos^{-1}\frac{-6}{\sqrt{40}}$ → $\theta = \cos^{-1}\frac{-6}{2\sqrt{10}}$

$\theta = \cos^{-1}\frac{-3}{\sqrt{10}}$

$\theta = 161.57°$

8. $\vec{v} = \frac{2}{3}\vec{i} + \frac{3}{2}\vec{j}$ and $\vec{w} = -\frac{5}{2}\vec{i} + \frac{4}{3}\vec{j}$

Dot Product $\frac{2}{3} \cdot \frac{-5}{2} + \frac{3}{2} \cdot \frac{4}{3} = -\frac{5}{3} + 2 = \frac{-5}{3} + \frac{6}{3} = \frac{1}{3}$

$\|\vec{v}\| = \sqrt{\frac{4}{9} + \frac{9}{4}} = \sqrt{\frac{97}{36}}$ $\|\vec{w}\| = \sqrt{\frac{25}{4} + \frac{16}{9}} = \sqrt{\frac{289}{36}}$

$\theta = \cos^{-1}\frac{1/3}{\sqrt{\frac{28033}{1296}}}$ $\theta = 85.89°$

For the following problems determine if the vectors are orthogonal (explain mathematically):

→ Dot Product = 0

9. $\vec{v} = \langle -8, -3 \rangle$ and $\vec{w} = \langle 3, -8 \rangle$

$-24 + 24 = 0$

yes - orthogonal (perpendicular) vectors

10. $\vec{v} = \langle 0, -7 \rangle$ and $\vec{w} = \langle 11, -2 \rangle$

$0 + 14 = 14$ Not orthogonal

11. $\vec{u} = \vec{i} + 2\vec{j}$ and $\vec{v} = 2\vec{i} - \vec{j}$

$2 - 2 = 0$

yes - orthogonal

12. $\vec{u} = 10\vec{i} - 2\vec{j}$ and $\vec{v} = 2\vec{i} + 9\vec{j}$

$20 - 18 = 2$ Not orthogonal

For the following problems find the dot product of the vectors given their magnitude and the angle in between the two vectors (round to the nearest hundredth): $\cos\theta = \frac{a \cdot b}{\|a\| \|b\|}$

13. *If* $||\vec{a}|| = 7$, $||\vec{b}|| = 8$, *and* $\theta = 155°$

$\cos 155° = \frac{a \cdot b}{7 \cdot 8}$

$\cos 155° = \frac{a \cdot b}{56}$ → $56 \cos 155 = a \cdot b$

$a \cdot b = -50.75$

14. *If* $||\vec{c}|| = 3$, $||\vec{d}|| = 11$, *and* $\theta = 65°$

$\cos 65° = \frac{c \cdot d}{3 \cdot 11}$

$33 \cos 65° = c \cdot d$ → $c \cdot d = 13.95$

15. *If* $||\vec{e}|| = 5$, $||\vec{f}|| = 7$, *and* $\theta = 102°$

$\cos 102° = \frac{e \cdot f}{5 \cdot 7}$

$35 \cos 102° = e \cdot f$ → $e \cdot f = -7.28$

16. *If* $||\vec{g}|| = 11$, $||\vec{h}|| = 2$, *and* $\theta = 14°$

$\cos 14° = \frac{g \cdot h}{11 \cdot 2}$

$22 \cos 14° = g \cdot h$ → $g \cdot h = 21.35$

For the following problems find the angle between the two vectors given their dot product. Use [0°, 360°). (round to the nearest hundredth): $\cos\theta = \frac{a \cdot b}{\|a\| \|b\|}$

17. *If* $||\vec{g}|| = 10$, $||\vec{h}|| = 20$, *and* $\vec{g} \cdot \vec{h} = -35$ *find* θ

$\cos\theta = \frac{-35}{10 \cdot 20}$

$\cos\theta = \frac{-35}{200}$ → $\theta = \cos^{-1}\left(-\frac{7}{40}\right)$

$\theta = 100.08°$

18. *If* $||\vec{v}|| = 12$, $||\vec{w}|| = 6$, *and* $\vec{v} \cdot \vec{w} = 67$ *find* θ

$\cos\theta = \frac{67}{12 \cdot 6}$

$\cos\theta = \frac{67}{72}$

$\theta = \cos^{-1}\left(\frac{67}{72}\right)$ → $\theta = 21.48°$

Answers:

1) 9 2) 26 3) 1/3 4) 0 5) 102.53° 6) 90° 7)161.57° 8) 85.89° 9) yes 10) no

11) yes 12) no 13) –50.75 14) 13.95 15) –7.28 16) 21.35 17) 100.08° 18) 21.48°