

Name: _____

Date: _____

Key

Independent and Dependent Events

Independent Events

- Event A occurring **does NOT** affect the probability of Event B occurring.

- $P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B)$

"And" means multiply

- A coin is tossed and a 6-sided die is rolled. Find the probability of landing on the head side of the coin and rolling a 3 on the die.

$P(\text{head and } 3)$

$$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$$

- A card is chosen at random from a deck of 52 cards. It is then **replaced** and a second card is chosen. What is the probability of choosing a jack and an eight?

$P(\text{Jack and } 8)$

$$\frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$$

- A jar contains 3 red, 5 green, 2 blue and 6 yellow marbles. A marble is chosen at random from the jar. After **replacing** it, a second marble is chosen. What is the probability of choosing a green and a yellow marble? Total # of marbles = 16

$P(\text{green and yellow})$

$$\frac{5}{16} \cdot \frac{6}{16} = \frac{15}{128}$$

- A school survey found that 9 out of 10 students like pizza. If three students are chosen at random with **replacement**, what is the probability that all three students like pizza? $P(\text{like and like and like})$

$$\frac{9}{10} \cdot \frac{9}{10} \cdot \frac{9}{10} = \frac{729}{1000}$$

Dependent Events

- Event A occurring **AFFECTS** the probability of Event B occurring.

- Usually you will see the words **"WITHOUT REPLACING."**

- $P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B|A)$

"And" means multiply

5. A jar contains 3 red, 5 green, 2 blue and 6 yellow marbles. A marble is chosen at random from the jar. A second marble is chosen **without replacing** the first one. What is the probability of choosing a green and a yellow marble?

$$P(\text{Green}) \cdot P(\text{yellow} | \text{Green})$$

$$\frac{5}{16} \cdot \frac{6}{15} = \boxed{\frac{1}{8}}$$

6. An aquarium contains 6 male goldfish and 4 female goldfish. You randomly select a fish from the tank, **do not replace** it, and then randomly select a second fish. What is the probability that both fish are male?

$$P(\text{male}) \cdot P(\text{male} | \text{male})$$

$$\frac{6}{10} \cdot \frac{5}{9} = \boxed{\frac{1}{3}}$$

7. A random sample of parts coming off a machine is done by an inspector. He found that 5 out of 100 parts are bad on average. If he were to do a new sample, what is the probability that he picks a bad part and then, picks another bad part if he **doesn't replace** the first? $P(\text{Bad}) \cdot P(\text{Bad} | \text{Bad})$

$$\frac{5}{100} \cdot \frac{4}{99} = \boxed{\frac{1}{495}}$$

How to Determine if 2 Events Are Independent:

- Substitute in what you know in to $P(A \cap B) = P(A) \cdot P(B)$ and check to see if left side equals right side.
 - If it's **equal**, then it's **independent**.
 - If it's **not equal**, then it's **not independent** (or dependent).

8. Let event M = taking a math class. Let event S = taking a science class. Then, M and S = taking a math class and a science class. Suppose $P(M) = 0.6$, $P(S) = 0.5$, and $P(M \text{ and } S) = 0.3$. Are M and S independent?

$$P(M \cap S) \stackrel{?}{=} P(M) \cdot P(S)$$

$$.3 = .6 \cdot .5$$

$$\boxed{.3 = .3 \quad \checkmark}$$

Taking a math class and taking a science class are independent of each other.

9. In a class, 60% of the students are female. 50% of all students in the class have long hair. 45% of the students are female and have long hair. Of the female students, 75% have long hair. Let F be the event that the student is female. Let L be the event that the student has long hair. One student is picked randomly. Are the events of being female and having long hair independent?

$$P(F \cap L) = P(F) \cdot P(L)$$

$$.45 \stackrel{?}{=} .6 \cdot .5$$

$$\boxed{.45 \neq .3}$$

Being a female and having long hair are not independent.