Parabolas – Part 1

Parabola

Every parabola has the property that any point on its graph is equidistant from a point called the focus and a line called the directrix.

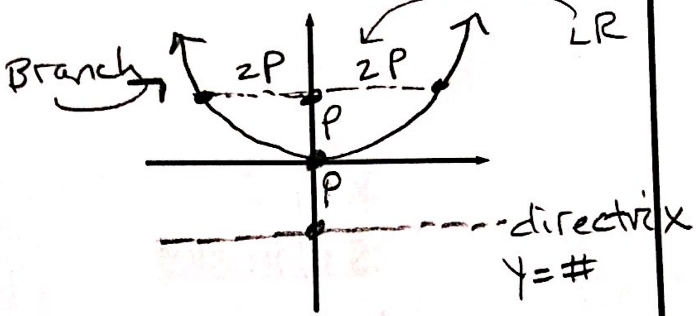
Vertical Parabola

$x^2 = 4py$

vertex $(0,0)$

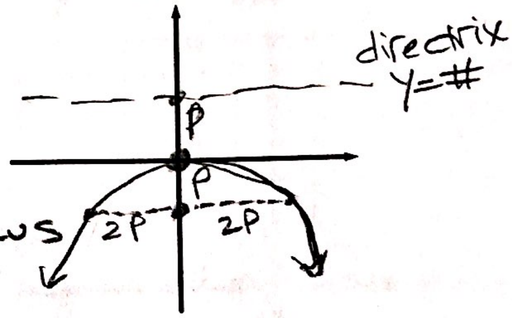
$(x-h)^2 = 4p(y-k)$

vertex (h,k)



x^2 → Positive 4P

x^2 → Negative 4P



P: distance from vertex to focus and vertex to directrix

$|4P|$ Lactus Rectum (LR) - width of parabola through the focus.

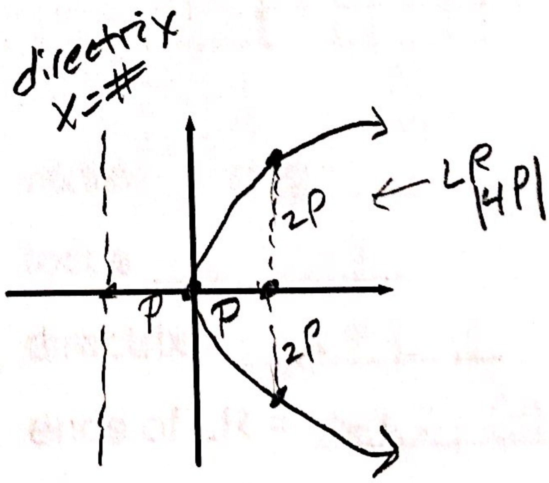
Horizontal Parabola

$y^2 = 4px$

vertex $(0,0)$

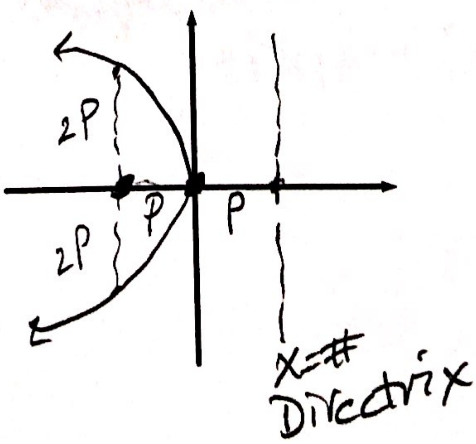
$(y-k)^2 = 4p(x-h)$

vertex (h,k)



y^2 → Positive 4P

y^2 → Negative 4P



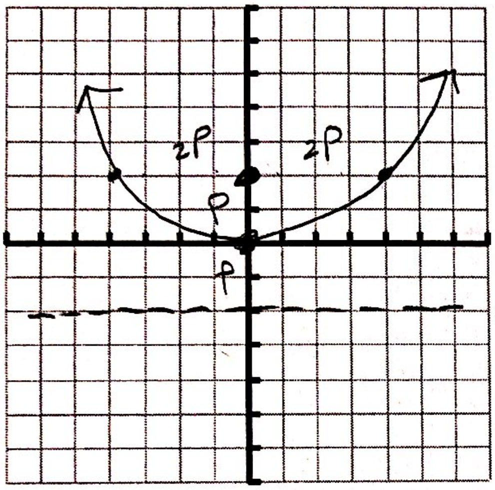
$x^2 = 4py$

Example 1:

$4p = 8$

$p = 2$

$$x^2 = 8y$$



x^2, Pos 4P

directrix
$y = -2$

vertex $(0, 0)$

focus $(0, 2)$

directrix = $y = -2$

ends of LR = $(4,2)(-4,2)$ $(\pm 4, 2)$

Length of LR = $|4p| = |8| = 8$

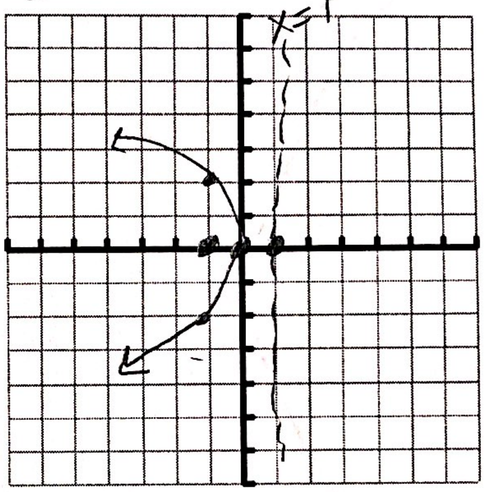
$y^2 = 4px$

Example 2:

$4p = -4$

$p = -1$

$$y^2 = -4x$$



y^2, Neg 4P

vertex $(0,0)$

focus $(-1,0)$

directrix = $x = 1$

ends of LR = $(-1,2)(-1,-2)$

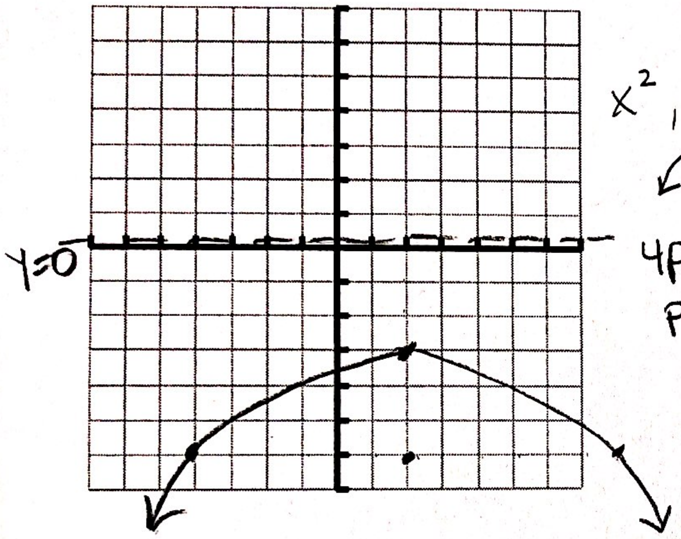
$(-1, \pm 2)$

Length of LR
$= |4p| = |-4| = 4$

$(x-h)^2 = 4p(y-k)$

Example 3:

$$(x-2)^2 = -12(y+3)$$



x^2, Neg 4p

$4p = -12$

$p = -3$

vertex $(2,-3)$

focus $(2,-6)$

directrix = $y=0$

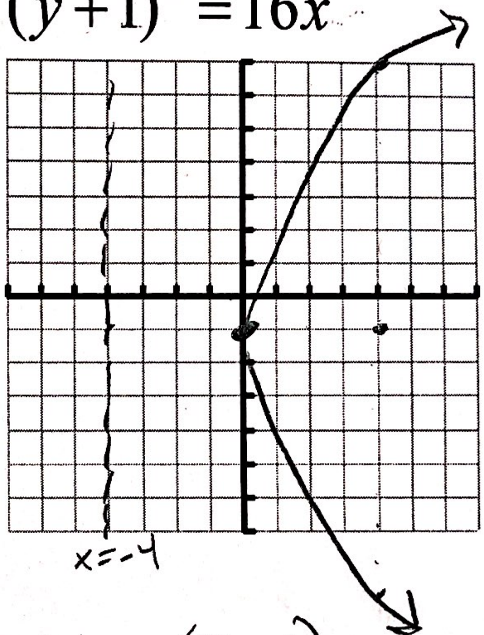
ends of LR = $(-4,-6)\ (8,-6)$

length of LR = $|4p| = |-12| = 12$

$(y-k)^2 = 4p(x-h)$

Example 4:

$$(y+1)^2 = 16x$$



y^2, pos 4p

$4p = 16$

$p = 4$

vertex $(0,-1)$

focus $(4,-1)$

directrix = $x=-4$

ends of LR = $(4,7)\ (4,-9)$

length of LR = $|4p| = |16| = 16$