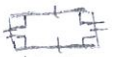
Geometry – Test Review

Unit 11

Name: Key

Date: _____ Period: ___

1. a. How can I prove a quadrilateral is a rectangle? 
   → opposite sides are congruent (use distance formula)
   → consecutive sides are perpendicular (use slope formula)

   b. How can I prove a quadrilateral is a square?
   → all sides are congruent (use distance formula) 
   → consecutive sides are perpendicular (use slope formula)

   c. How can I prove a quadrilateral is a trapezoid?
   → 1 pair of opposite sides parallel (+1 pair not ||) 
   (use slope formula)

   d. How can I prove a quadrilateral is an isosceles trapezoid?
   → 1 pair of opposite sides parallel (+1 pair not ||) 
   → 1 pair of opposite sides congruent (use slope + distance formula)

2. a. Write the equation of this circle in **general** form: $y^2 + x^2 - 1 = -4x + 4y$

   $x^2+y^2+4x-4y-1=0$

   b. Now write that equation in **standard** form. $x^2+4x+4+y^2-4y+4=1+4+4$

   $(x+2)^2+(y-2)^2=9$

3. What is the slope of a line parallel and perpendicular to a line with a slope of -5/6?

   $m_{\parallel} = -5/6$

   $m_{\perp} = 6/5$

4. A side of a **square** has coordinates (-3, 2) and (2, 5). What is the perimeter of the square?

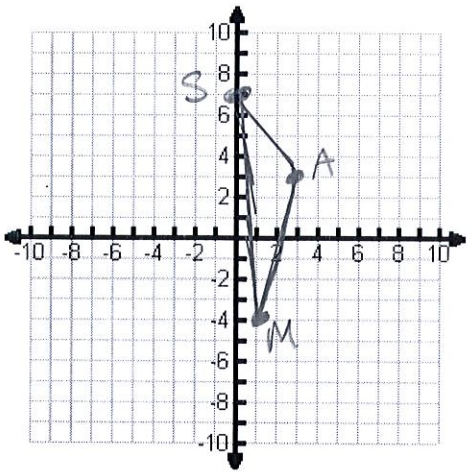
   Use distance formula to find the length of 1 side of square.

   $d_{side} = \sqrt{(2-3)^2+(5-2)^2} = \sqrt{(5)^2+(3)^2} = \sqrt{25+9} = \sqrt{34}$

   Perimeter $= \sqrt{34}(4) = \boxed{4\sqrt{34}}$

no sides congruent — 2 sides congruent — all sides congruent

Classify the following triangle by sides (scalene, isosceles, equilateral) and determine if it is a right triangle if the coordinates of the vertices are S(0, 7), A(3, 3), and M(1, -4).



Distance (length)

$SA: \sqrt{(3-0)^2+(3-7)^2} = \sqrt{3^2+(-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$AM: \sqrt{(1-3)^2+(-4-3)^2} = \sqrt{(-2)^2+(-7)^2} = \sqrt{4+49} = \sqrt{53}$

$MS: \sqrt{(1-0)^2+(-4-7)^2} = \sqrt{1^2+(-11)^2} = \sqrt{1+121} = \sqrt{122}$

Slope

$m_{SA} = \frac{3-7}{3-0} = \frac{-4}{3}$

$m_{AM} = \frac{-4-3}{1-3} = \frac{-7}{-2} = \frac{7}{2}$

$m_{MS} = \frac{-4-7}{1-0} = \frac{-11}{1} = -11$

6. Determine if the following points are inside, outside, or on the circle $(x-2)^2 + (y+3)^2 = 220$?

a. (-8, 8) outside

$(-8-2)^2+(8+3)^2 = 220$

$100+121 = 220$

$221 > 220$

b. (4, -13) Inside

$(4-2)^2+(-13+3)^2 = 220$

$4+100 = 220$

$104 < 220$

c. (-9, -2) Inside

$(-9-2)^2+(-2+3)^2 = 220$

$121+1 = 220$

$122 < 220$

7. Find the slope, distance, and midpoint of (21, 5) and (5, 7).

Slope: $-1/8$

$m = \frac{7-5}{5-21} = \frac{2}{-16} = \frac{-1}{8}$

Distance: $2\sqrt{65}$

$\sqrt{(5-21)^2+(7-5)^2}$

$\sqrt{256+4}$

$\sqrt{260} = \sqrt{4}\sqrt{65}$

Midpoint: (13, 6)

$\left(\frac{21+5}{2}, \frac{5+7}{2}\right)$

$\left(\frac{26}{2}, \frac{12}{2}\right)$

8. Find the perpendicular slope of (12, 4) and (18, 12).

$m = \frac{12-4}{18-12} = \frac{8}{6} = \frac{4}{3}$

$m_{\perp} = -\frac{3}{4}$

9. Write the equation of this circle in general form: $(x-1)^2 + (y+2)^2 = 9$

$(x-1)(x-1)+(y+2)(y+2) = 9$

$x^2-x-x+1+y^2+2y+2y+4-9 = 0$

$x^2+y^2-2x+4y-4 = 0$

. Write the equation of this circle in standard form: $y^2 + 8y = -12 - x^2$

Then find the center and radius.

$x^2 + y^2 + 8y + 16 = -12 + 16$

$x^2 + (y+4)^2 = 4$

Equation: $x^2 + (y+4)^2 = 4$

Center: $(0, -4)$

Radius: 2

11. What is the equation of a circle that has a center (h, k) $(8, 12)$ and a point on the circle at (2, -4)?

$(x-8)^2 + (y-12)^2 = \sqrt{292}^2$

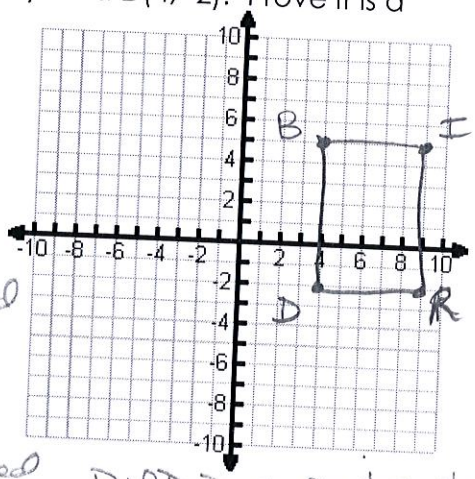
$\boxed{(x-8)^2 + (y-12)^2 = 292}$

$d(\text{radius}) = \sqrt{(2-8)^2 + (-4-12)^2}$

$= \sqrt{36 + 256} = \sqrt{292}$

12. The coordinates for quadrilateral BIRD are B(4, 5), I(9, 5), R(9, -2), and D(4, -2). Prove it is a rectangle. *use distance formula for length

*use slope formula for right angles



Length

BI: $\sqrt{(9-4)^2 + (5-5)^2} = \sqrt{25} = 5$

IR: $\sqrt{(9-9)^2 + (-2-5)^2} = \sqrt{49} = 7$

RD: $\sqrt{(4-9)^2 + (-2-(-2))^2} = \sqrt{25} = 5$

DB: $\sqrt{(4-4)^2 + (-2-5)^2} = \sqrt{49} = 7$

Slope

$\frac{5-5}{9-4} = \frac{0}{5} = 0$

$\frac{-2-5}{9-9} = \frac{-7}{0} = \text{undefined}$

$\frac{-2-(-2)}{4-9} = \frac{0}{-5} = 0$

$\frac{-2-5}{4-4} = \frac{-7}{0} = \text{undefined}$

BIRD is a rectangle because $\overline{BI} \cong \overline{DR}$, $\overline{DB} \cong \overline{IR}$, $\overline{BI} \perp \overline{IR}$, $\overline{IR} \perp \overline{RD}$, $\overline{RD} \perp \overline{DB}$, $\overline{BI} \cong \overline{DB}$

13. Now prove that #12 is NOT a square.

$\overline{BI} \ncong \overline{IR}$ $\overline{IR} \ncong \overline{RD}$, $\overline{RD} \ncong \overline{DB}$, $\overline{DB} \ncong \overline{BI}$

All sides are not congruent so BIRD is not a square

14. What is the equation of a circle that has endpoints of a diameter at (-2, 1) and (5, 5)?

1 - Find midpoint

$\left(\frac{-2+5}{2}, \frac{1+5}{2}\right) = \left(\frac{3}{2}, 3\right)$

Center = (1.5, 3)

2- Find distance between center and 1 point on circle

$d = \sqrt{(1.5-2)^2 + (3-1)^2}$

$= \sqrt{3.5^2 + 2^2}$

Radius = $\sqrt{16.25}$

$(x-h)^2 + (y-k)^2 = r^2$

$\boxed{(x-1.5)^2 + (y-3)^2 = 16.25}$

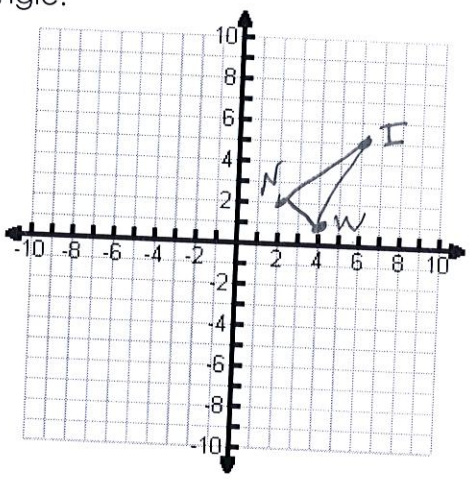
15. Prove that the triangle W(4, 1), I(6, 5), and N(2, 2) is a right triangle.

$m_{WI} = \frac{5-1}{6-4} = \frac{4}{2} = 2$

$m_{IN} = \frac{2-5}{2-6} = \frac{-3}{-4} = \frac{3}{4}$

$m_{WN} = \frac{2-1}{2-4} = \frac{1}{-2}$

WIN is a right triangle because $\overline{WI} \perp \overline{WN}$.



16. Prove that the quadrilateral SWAG is an isosceles trapezoid. S(3, 7), W(-3, 7), A(6, -2), and G(6, 4)

SW: $\sqrt{(-3-3)^2+(7-7)^2} = \sqrt{36} = 6$ (congruent sides)

WA: $\sqrt{(6-(-3))^2+(-2-7)^2} = \sqrt{81+81} = \sqrt{162} = 9\sqrt{2}$

AG: $\sqrt{(6-6)^2+(4-(-2))^2} = \sqrt{36} = 6$ (congruent sides)

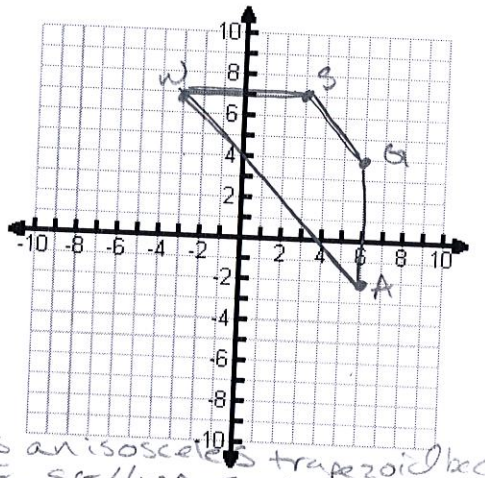
GS: $\sqrt{(6-3)^2+(4-7)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$m_{SW} = \frac{7-7}{-3-3} = \frac{0}{-6} = 0$ (parallel sides)

$m_{WA} = \frac{-2-7}{6-(-3)} = \frac{-9}{9} = -1$

$m_{AG} = \frac{4-(-2)}{6-6} = \frac{6}{0}$ = undefined (parallel sides)

$m_{SG} = \frac{4-7}{6-3} = \frac{-3}{3} = -1$



SWAG is an isosceles trapezoid because $\overline{SW} \cong \overline{AG}$, $\overline{SG} \parallel \overline{WA}$, $\overline{SW} \not\parallel \overline{AG}$.

Write the equation for a line that is *parallel* to the given line and contains the following points.

17. x = -7 Contains the point (3, 8)

↳ vertical line

→ use x value for equation

equation: x = 3

18. $\frac{3y}{3} = \frac{x}{3} + \frac{15}{3}$ Contains the point (-3, 10) (x, y)

$y = \frac{1}{3}x + 5$ $m = \frac{1}{3}$ $y = mx + b$

$10 = (\frac{1}{3})(-3) + b$

$10 = -1 + b$

$+1 \quad +1$

$11 = b$

equation: $y = \frac{1}{3}x + 11$

Write the equation for a line that is *perpendicular* to the given line and contains the following points.

19. 5x – 3y = 6 Contains the point (5, -6)

$-5x \quad -5x$

$\frac{-3y}{-3} = \frac{-5x}{-3} + \frac{6}{-3}$

$y = \frac{5}{3}x - 2$ → $m = \frac{5}{3}$ $\perp m = -\frac{3}{5}$

$y = mx + b$

$-6 = (-\frac{3}{5})(5) + b$

$-6 = -3 + b$

$+3 \quad +3$

$-3 = b$

equation: $y = -\frac{3}{5}x - 3$

20. y = 4 Contains the point (-5, -7)

↳ horizontal line

→ use the x-value

equation: x = -5