Show What Ya Know! – Proofs & Circles

Name: Key
Date: ________ Period: ________

1. Prove that the quadrilateral ABCD with vertices A(2,1) B(1,3) C(-5,0) D(-4,-2) is a rectangle.

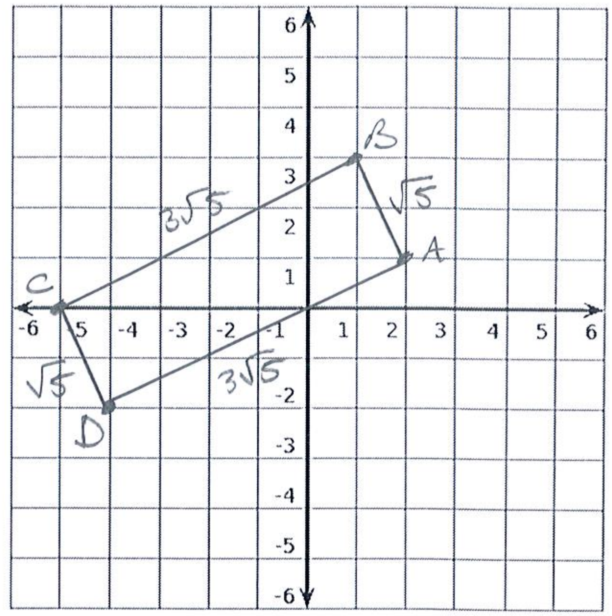
SIDE LENGTHS: (use distance)

AB: $\sqrt{(1-2)^2+(3-1)^2} = \sqrt{(-1)^2+(2)^2} = \sqrt{1+4} = \sqrt{5}$

BC: $\sqrt{(-5-1)^2+(0-3)^2} = \sqrt{(-6)^2+(-3)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

CD: $\sqrt{(-4-{-5})^2+(-2-0)^2} = \sqrt{(1)^2+(-2)^2} = \sqrt{1+4} = \sqrt{5}$

AD: $\sqrt{(-4-2)^2+(-2-1)^2} = \sqrt{(-6)^2+(-3)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$



SLOPES:

$m_{AB} = \frac{3-1}{1-2} = \frac{2}{-1} = -2$

$m_{BC} = \frac{0-3}{-5-1} = \frac{-3}{-6} = \frac{1}{2}$

$m_{CD} = \frac{-2-0}{-4-5} = \frac{-2}{1} = -2$

$m_{AD} = \frac{-2-1}{-4-2} = \frac{-3}{-6} = \frac{1}{2}$

STATEMENT: ABCD is a rectangle because $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{AD}$, $\overline{AB} \perp \overline{BC}$, $\overline{BC} \perp \overline{CD}$, $\overline{CD} \perp \overline{AD}$, $\overline{AB} \perp \overline{AD}$

2. Find the equation of a circle given the endpoints of the diameter are (11, -2) and (7,-8).

Center: (9, -5)

Center (midpoint formula) $= \left(\frac{11+7}{2}, \frac{-2+-8}{2}\right) = \boxed{(9,-5)}$

Radius: $\sqrt{13}$

Radius (distance formula) $= \sqrt{(9-11)^2+(-5-{-2})^2} = \sqrt{(-2)^2+(-3)^2}$
$= \sqrt{4+9} = \sqrt{13}$

Equation of circle: $(x-9)^2+(y+5)^2=13$

$r=\sqrt{13}$
$r^2=\sqrt{13}^2$

3. Convert the general form equation of a circle to standard form: $x^2 + y^2 - 8x + 18y + 72 = 0$

Equation of circle: $(x-4)^2+(y+9)^2=25$

$x^2-8x+16+y^2+18y+81=-72+16+81$
$(x-4)^2+(y+9)^2=25$

Center: (4, -9)

Radius: 5