Geometry Name: Key
Coordinate Proofs – Trapezoids & Triangles Date: ____________

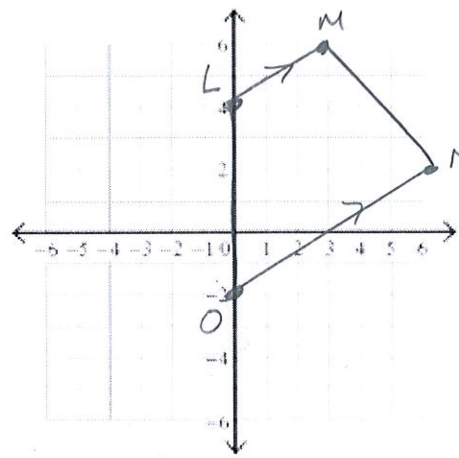
TRAPEZOID: A trapezoid is a quadrilateral with ONE pair of opposite sides that are parallel (and the other pair of sides are NOT PARALLEL).

Proving a Quadrilateral is a Trapezoid

Example 1: Prove that quadrilateral LMNO is a trapezoid with coordinates L(0, 4), M(3, 6), N(6, 2) and O(0, -2).

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Step 1: Plot the vertices on the coordinate plane.



Step 2: Find the key word. (Trapezoid)

Find the ***slope*** of each side.

m_{LM}: 2/3 $\frac{6-4}{3-0} = \frac{2}{3}$ m_{MN}: -4/3 $\frac{2-6}{6-3} = \frac{-4}{3}$

m_{NO}: 2/3 $\frac{-2-2}{0-6} = \frac{-4}{-6}$ m_{LO}: undefined $\frac{-2-4}{0-0} = \frac{-6}{0}$

Step 3: CONCLUSION

LMNO is a trapezoid because $\overline{LM} \parallel \overline{NO}$, $\overline{MN} \nparallel \overline{LO}$.

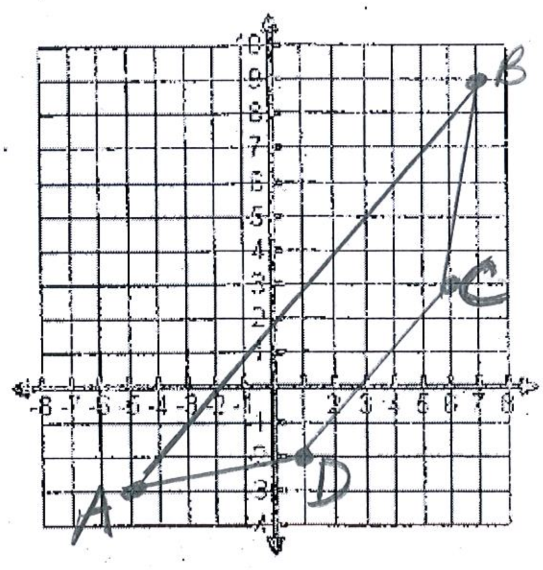
ISOSCELES TRAPEZOID: An isosceles trapezoid is a trapezoid with ONE pair of opposite sides that are congruent.

→ exactly 1 pair of parallel sides.
→ exactly 1 pair of congruent sides

Proving a Quadrilateral is an Isosceles Trapezoid

Example 2: Prove that quadrilateral ABCD is an isosceles trapezoid with coordinates A(-5, -3), B(7, 9), C(6, 3) and D(1, -2).

Step 1: Plot the vertices on the coordinate plane.



Step 2: Find the key word. (isosceles trapezoid)

Find the ***slope*** of each side.

m_{AB} = 1 $\frac{9-3}{7-5} = \frac{12}{12}$ m_{BC} = 6 $\frac{3-9}{6-7} = \frac{-6}{-1}$

m_{CD} = 1 $\frac{-2-3}{1-6} = \frac{-5}{-5}$ m_{AD} = $\frac{1}{6}$ $\frac{-2-3}{1-5} = \frac{1}{6}$

Find the ***length*** of each side.

AB: $12\sqrt{2}$ $\sqrt{(7-5)^2+(9-3)^2}$ $\sqrt{144+144}$ $\sqrt{288} = 12\sqrt{2}$

BC: $\sqrt{37}$ $\sqrt{(6-7)^2+(3-9)^2}$ $\sqrt{1+36} = \sqrt{37}$

CD: $5\sqrt{2}$ $\sqrt{(6-1)^2+(3-2)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$

AD: $\sqrt{37}$ $\sqrt{(1-5)^2+(-2-3)^2} = \sqrt{36+1} = \sqrt{37}$

Step 3: CONCLUSION

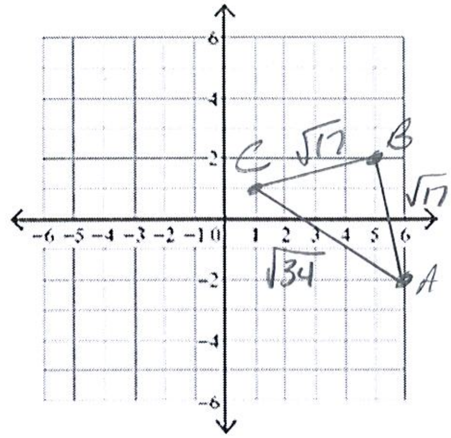
ABCD is an isosceles trapezoid because $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \nparallel \overline{AD}$, $\overline{BC} \cong \overline{AD}$.

Proving a Triangle is SCALENE, ISOSCELES, OR EQUILATERAL

Example 3: Classify triangle ABC as scalene, isosceles, or equilateral.
A(6, -2), B(5, 2), and C(1, 1).

no sides equal — 2 sides equal — 3 sides equal

Step 1: Plot the vertices on the coordinate plane.



Step 2: Find the key word(s). (Scalene, isosceles or equilateral)

Find the ***length*** of each side. $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

AB: $\sqrt{17}$ $\sqrt{(5-6)^2+(2-{}^-2)^2}=\sqrt{1^2+4^2}=\sqrt{1+16}=\sqrt{17}$

BC: $\sqrt{17}$ $\sqrt{(1-5)^2+(1-2)^2}=\sqrt{(-4)^2+(-1)^2}=\sqrt{16+1}=\sqrt{17}$

CA: $\sqrt{34}$ $\sqrt{(1-6)^2+(1-{}^-2)^2}=\sqrt{(-5)^2+(3)^2}=\sqrt{25+9}=\sqrt{34}$

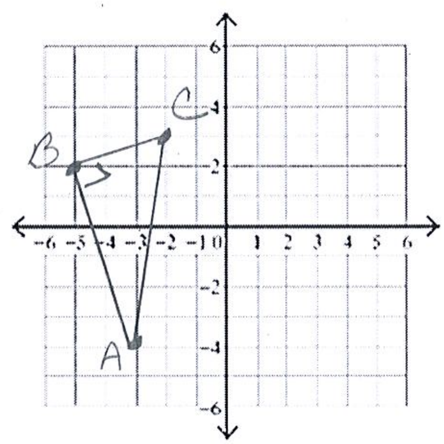
Step 3: CONCLUSION

ABC is an isosceles triangle because $\overline{AB}\cong\overline{BC}$.

Proving a Triangle is a Right Triangle

Example 4: Given the coordinates A(-3, -4), B(-5, 2), and C(-2, 3), prove that ABC is a right triangle.

Step 1: Plot the vertices on the coordinate plane.



Step 2: Find the key word. (Right triangle)

Find the ***slope*** of each side.

m_{AB} = -3 $\quad \frac{2-{}^-4}{-5-{}^-3}=\frac{6}{-2}=-3$

m_{BC} = $\frac{1}{3}$ $\quad \frac{3-2}{-2-{}^-5}=\frac{1}{3}$

m_{CA} = 7 $\quad \frac{3-{}^-4}{-2-{}^-3}=\frac{7}{1}$

Step 3: CONCLUSION

ABC is a right triangle because $\overline{AB}\perp\overline{BC}$.