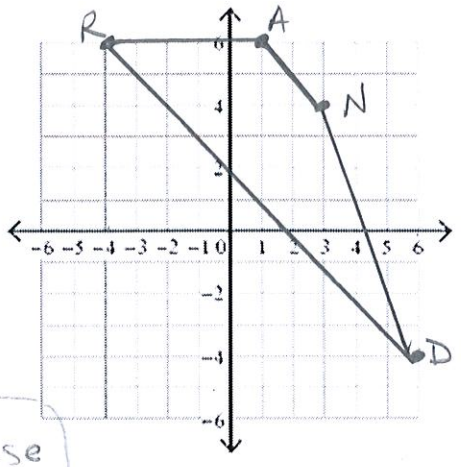
Geometry Name: Key

Classwork/Homework: Coordinate Proofs Date: _____

Proving a Quadrilateral is a Trapezoid

1. Prove that quadrilateral RAND is a trapezoid with coordinates R(-4, 6), A(1, 6), N(3, 4) and D(6, -4).



Slope

$m_{RA} = \frac{6-6}{1-4} = \frac{0}{5} = 0$

$m_{AN} = \frac{4-6}{3-1} = \frac{-2}{2} = -1$

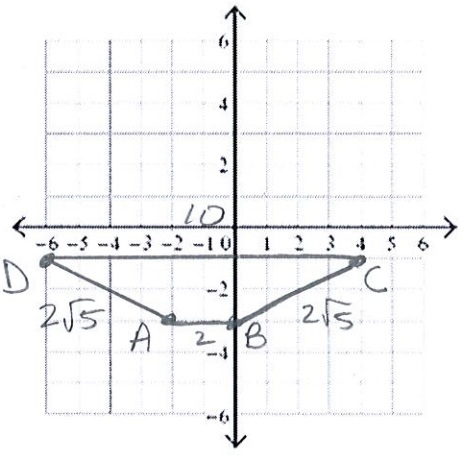
$m_{ND} = \frac{-4-4}{6-3} = \frac{-8}{3}$

$m_{RD} = \frac{-4-6}{6-4} = \frac{-10}{10} = -1$

Conclusion: RAND is a trapezoid because $\overline{AN} \parallel \overline{RD}$, $\overline{RA} \nparallel \overline{ND}$

Proving a Quadrilateral is an Isosceles Trapezoid

2. Prove that quadrilateral ABCD is an isosceles trapezoid with coordinates A(-2, -3), B(0, -3), C(4, -1) and D(-6, -1).



Slope

$m_{AB} = \frac{-3-3}{0-2} = \frac{0}{2} = 0$

$m_{BC} = \frac{-1-3}{4-0} = \frac{2}{4} = \frac{1}{2}$

$m_{CD} = \frac{-1-1}{4-6} = \frac{0}{10} = 0$

$m_{DA} = \frac{-3-1}{-2-6} = \frac{-2}{4} = -\frac{1}{2}$

Length

$AB: \sqrt{(0-2)^2+(-3-3)^2} = \sqrt{4+0} = \sqrt{4} = 2$

$BC: \sqrt{(4-0)^2+(-1-3)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

$CD: \sqrt{(-6-4)^2+(-1+1)^2} = \sqrt{100+0} = 10$

$DA: \sqrt{(-2-6)^2+(-3-1)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

Conclusion:

ABCD is an isosceles trapezoid because $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \nparallel \overline{DA}$

$\overline{BC} \cong \overline{DA}$

Proving Triangles

3. Classify the triangle as scalene, isosceles, or equilateral. Determine if it is also a right triangle.

a. (1, 4), (4, 5), (5, 2) — A, B, C

Classify:

$AB = \sqrt{(4-1)^2+(5-4)^2} = \sqrt{9+1} = \sqrt{10}$

$BC = \sqrt{(5-4)^2+(2-5)^2} = \sqrt{1+9} = \sqrt{10}$

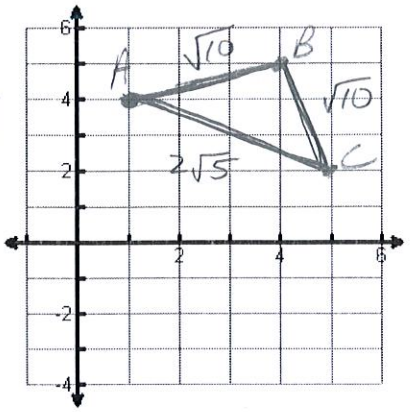
$CA = \sqrt{(5-1)^2+(2-4)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

2 equal sides = isosceles

Right triangle? yes

$m_{AB} = \frac{5-4}{4-1} = \frac{1}{3}$ $m_{AC} = \frac{4-2}{1-5} = \frac{2}{-4} = -\frac{1}{2}$

$m_{BC} = \frac{2-5}{5-4} = \frac{-3}{1} = -3$



b. (0, –2), (0, 2), (4, 0) — A, B, C

Classify:

$AB = \sqrt{(0-0)^2+(2-(-2))^2} = \sqrt{4^2} = \sqrt{16} = 4$

$BC = \sqrt{(4-0)^2+(0-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

$CA = \sqrt{(4-0)^2+(0-(-2))^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

Isosceles

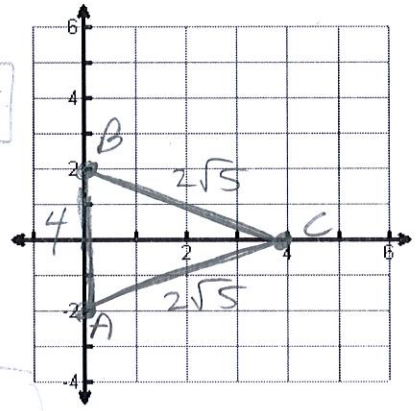
Right triangle?

$m_{AB} = \frac{2-(-2)}{0-0} = \frac{4}{0}$ = undefined

$m_{BC} = \frac{0-2}{4-0} = \frac{-2}{4} = -\frac{1}{2}$

$m_{AC} = \frac{0-(-2)}{4-0} = \frac{2}{4} = \frac{1}{2}$

No sides are perpendicular so not a right triangle.



c. (0, 0), (2, 0), (4, –3) — A, B, C

Classify:

$AB = \sqrt{(2-0)^2+(0-0)^2} = \sqrt{4} = 2$

$BC = \sqrt{(4-2)^2+(-3-0)^2} = \sqrt{4+9} = \sqrt{13}$

$CA = \sqrt{(4-0)^2+(-3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5$

No sides equal so scalene

Right triangle?

$m_{AB} = \frac{0-0}{2-0} = \frac{0}{2} = 0$

$m_{BC} = \frac{-3-0}{4-2} = \frac{-3}{2}$

$m_{CA} = \frac{4-0}{-3-0} = \frac{4}{-3}$

No slopes are opposite reciprocals so not a right triangle.