

Formulas and the Coordinate Plane

Formula

- Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

When to Use it

To determine whether:

➤ sides are congruent

To determine whether:

➤ opposite sides are parallel

➤ consecutive sides are perpendicular

Proving that a Quadrilateral is a Rectangle

Prove that G(1, 1), H(5, 3), I(4, 5), and J(0, 3) is a Rectangle

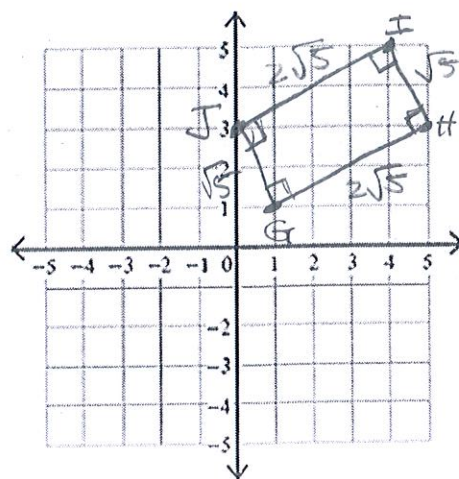
- Key Word: Rectangle
- Step 1: Calculate distances of all four sides to show that opposite sides are congruent.

$$GH = \sqrt{(5-1)^2 + (3-1)^2} = \sqrt{4^2 + 2^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$HI = \sqrt{(4-5)^2 + (5-3)^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5}$$

$$IJ = \sqrt{(0-4)^2 + (3-5)^2} = \sqrt{(-4)^2 + (-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$JG = \sqrt{(0-1)^2 + (3-1)^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5}$$



- Step 2: Calculate the slopes of all four sides to show that consecutive sides are perpendicular.

$$m_{GH} = \frac{3-1}{5-1} = \frac{2}{4} = \frac{1}{2}$$

$$m_{IJ} = \frac{3-5}{0-4} = \frac{-2}{-4} = \frac{1}{2}$$

$$m_{HI} = \frac{5-3}{4-5} = \frac{2}{-1} = -2$$

$$m_{GJ} = \frac{3-1}{0-1} = \frac{2}{-1} = -2$$

- Statement:

GHIJ is a rectangle because $\overline{JI} \cong \overline{GH}$, $\overline{JG} \cong \overline{IH}$
 $\overline{GH} \perp \overline{HI}$, $\overline{IJ} \perp \overline{GJ}$, $\overline{HI} \perp \overline{IJ}$, $\overline{HG} \perp \overline{JG}$

Proving that a Quadrilateral is a Square

Prove that a quadrilateral with vertices $A(-1, 0)$, $B(3, 3)$, $C(6, -1)$ and $D(2, -4)$ is a square.

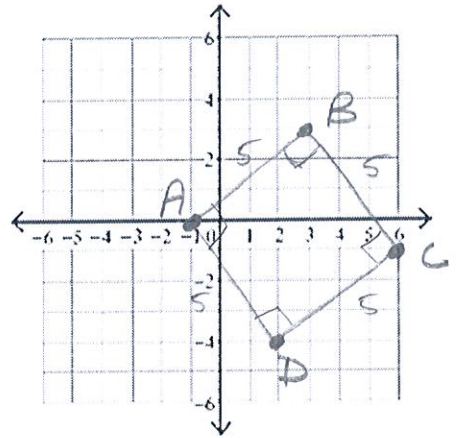
- Key Word: Square
- Step 1: Calculate distances of all four sides to show that all sides are congruent.

$$AB = \sqrt{(3-(-1))^2 + (3-0)^2} = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$BC = \sqrt{(6-3)^2 + (-1-3)^2} = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$CD = \sqrt{(2-6)^2 + (-4-(-1))^2} = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$DA = \sqrt{(2-(-1))^2 + (-4-0)^2} = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$



- Step 2: Calculate the slopes of all four sides to show that consecutive sides are perpendicular.

$$m_{AB} = \frac{3-0}{3-(-1)} = \frac{3}{4}$$

$$m_{CD} = \frac{-4-(-1)}{2-6} = \frac{-3}{-4} = \frac{3}{4}$$

$$m_{BC} = \frac{-1-3}{6-3} = \frac{-4}{3}$$

$$m_{DA} = \frac{-4-0}{2-(-1)} = \frac{-4}{3}$$

- Statement:

ABCD is a square because $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$
 $\overline{AB} \perp \overline{BC}$, $\overline{CD} \perp \overline{DA}$, $\overline{AB} \perp \overline{DA}$, $\overline{BC} \perp \overline{CD}$