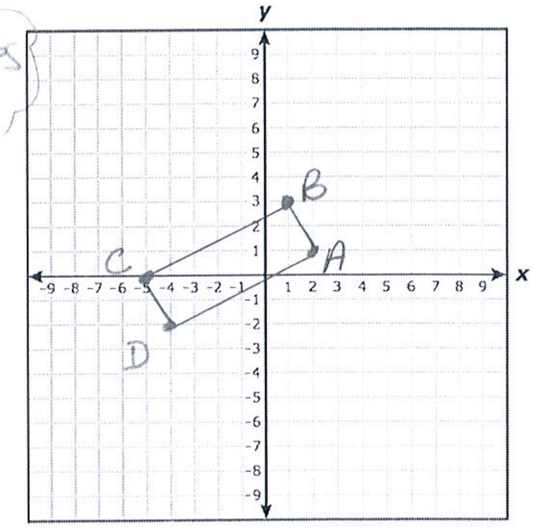
Geometry
Classwork/Homework – Rectangles & Squares

Name: Key
Date: ______________

1. Prove that a quadrilateral ABCD with the vertices A(2, 1), B(1, 3), C(-5, 0), and D(-4, -2) is a rectangle.

① Plot points
② find length of sides
③ find slope of sides
④ Statement

Key Word: rectangle



SIDE LENGTHS:

AB: $\sqrt{(1-2)^2+(3-1)^2} = \sqrt{(-1)^2+(2)^2} = \sqrt{1+4} = \sqrt{5}$

BC: $\sqrt{(-5-1)^2+(0-3)^2} = \sqrt{(-6)^2+(-3)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

CD: $\sqrt{(-4-^-5)^2+(-2-0)^2} = \sqrt{1^2+(-2)^2} = \sqrt{1+4} = \sqrt{5}$

AD: $\sqrt{(-4-2)^2+(-2-1)^2} = \sqrt{(-6)^2+(-3)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

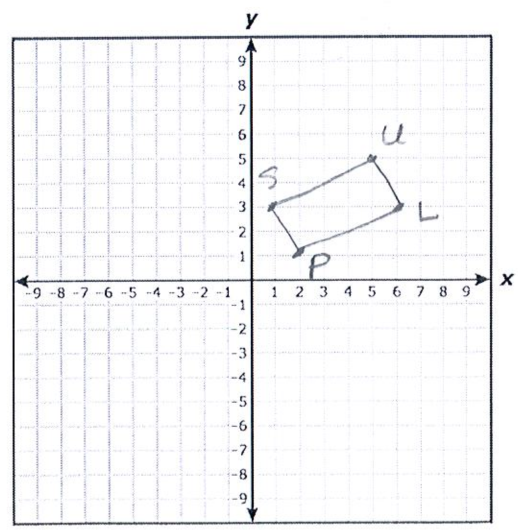
SLOPES:

$m_{AB} = \frac{3-1}{1-2} = \frac{2}{-1} = -2$ $\quad$ $m_{CD} = \frac{-2-0}{-4-^-5} = \frac{-2}{1} = -2$

$m_{BC} = \frac{0-3}{-5-1} = \frac{-3}{-6} = \frac{1}{2}$ $\quad$ $m_{AD} = \frac{-2-1}{-4-2} = \frac{-3}{-6} = \frac{1}{2}$

Statement: ABCD is a rectangle because $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{AD}$, $\overline{AB} \perp \overline{BC}$, $\overline{CD} \perp \overline{AD}$, $\overline{AB} \perp \overline{AD}$, $\overline{BC} \perp \overline{CD}$

2. Prove that a quadrilateral PLUS with the vertices P(2, 1), L(6, 3), U(5, 5), and S(1, 3) is a rectangle.



Side lengths

$PL = \sqrt{(6-2)^2+(3-1)^2} = \sqrt{4^2+2^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

$LU = \sqrt{(5-6)^2+(5-3)^2} = \sqrt{(-1)^2+(2)^2} = \sqrt{1+4} = \sqrt{5}$

$US = \sqrt{(1-5)^2+(3-5)^2} = \sqrt{(-4)^2+(-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

$SP = \sqrt{(1-2)^2+(3-1)^2} = \sqrt{(-1)^2+(2)^2} = \sqrt{1+4} = \sqrt{5}$

Slopes

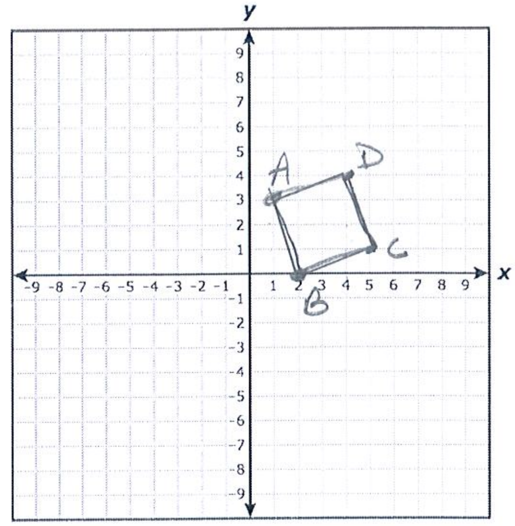
$m_{PL} = \frac{3-1}{6-2} = \frac{2}{4} = \frac{1}{2}$ $\quad$ $m_{US} = \frac{3-5}{1-5} = \frac{-2}{-4} = \frac{1}{2}$

$m_{LU} = \frac{5-3}{5-6} = \frac{2}{-1} = -2$ $\quad$ $m_{SP} = \frac{3-1}{1-2} = \frac{2}{-1} = -2$

Statement

PLUS is a rectangle because $\overline{PL} \cong \overline{US}$, $\overline{LU} \cong \overline{SP}$, $\overline{PL} \perp \overline{LU}$, $\overline{US} \perp \overline{SP}$, $\overline{PL} \perp \overline{SP}$, $\overline{LU} \perp \overline{US}$

3. Prove that a quadrilateral ABCD with the vertices A(1, 3), B(2, 0), C(5, 1), and D(4, 4) is a square.



Key Word: Square

SIDE LENGTHS:

AB: $\sqrt{(2-1)^2+(0-3)^2} = \sqrt{1^2+(-3)^2} = \sqrt{1+9} = \sqrt{10}$

BC: $\sqrt{(5-2)^2+(1-0)^2} = \sqrt{3^2+1^2} = \sqrt{9+1} = \sqrt{10}$

CD: $\sqrt{(4-5)^2+(4-1)^2} = \sqrt{(-1)^2+3^2} = \sqrt{1+9} = \sqrt{10}$

AD: $\sqrt{(4-1)^2+(4-3)^2} = \sqrt{3^2+1^2} = \sqrt{9+1} = \sqrt{10}$

SLOPES:

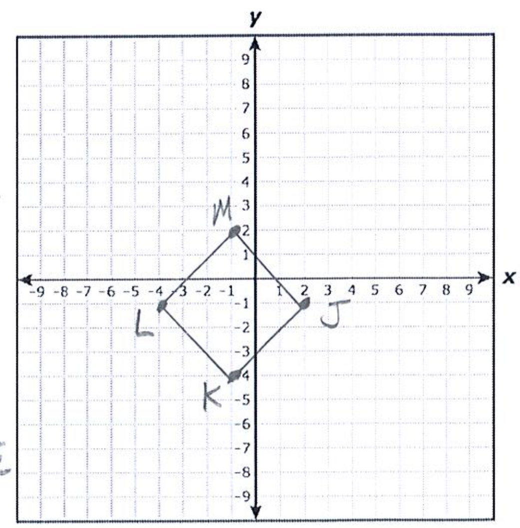
$m_{AB} = \frac{0-3}{2-1} = \frac{-3}{1} = -3$ $\quad m_{CD} = \frac{4-1}{4-5} = \frac{3}{-1} = -3$

$m_{BC} = \frac{1-0}{5-2} = \frac{1}{3}$ $\quad m_{AD} = \frac{4-3}{4-1} = \frac{1}{3}$

Statement:

ABCD is a square because $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$, $\overline{AB} \perp \overline{BC}$, $\overline{CD} \perp \overline{AD}$, $\overline{AB} \perp \overline{AD}$, $\overline{BC} \perp \overline{CD}$

4. Prove that a quadrilateral ABCD with the vertices J(2, -1), K(-1, -4), L(-4, -1), and M(-1, 2) is a square.



Side lengths

$JK = \sqrt{(-1-2)^2+(-4-{}^-1)^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$KL = \sqrt{(-4-{}^-1)^2+(-1-{}^-4)^2} = \sqrt{(-3)^2+(3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$LM = \sqrt{(-1-{}^-4)^2+(2-{}^-1)^2} = \sqrt{3^2+3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$MJ = \sqrt{(-1-2)^2+(2-{}^-1)^2} = \sqrt{(-3)^2+(3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Slopes

$m_{JK} = \frac{-4-{}^-1}{-1-2} = \frac{-3}{-3} = 1$

$m_{KL} = \frac{-1-{}^-4}{-4-{}^-1} = \frac{3}{-3} = -1$

$m_{LM} = \frac{2-{}^-1}{-1-{}^-4} = \frac{3}{3} = 1$

$m_{MJ} = \frac{2-{}^-1}{-1-2} = \frac{3}{-3} = -1$

Statement

JKLM is a square because $\overline{JK} \cong \overline{KL} \cong \overline{LM} \cong \overline{MJ}$, $\overline{JK} \perp \overline{KL}$, $\overline{KL} \perp \overline{LM}$, $\overline{LM} \perp \overline{MJ}$, $\overline{JK} \perp \overline{MJ}$