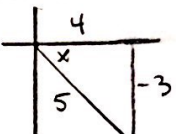


$$\frac{\frac{3\pi}{2}}{2} = \frac{3\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$$

## Warmup 4: Double and Half Angle Identities

Given  $\cos x = \frac{4_{\text{adj}}}{5_{\text{hyp}}}$  and  $\frac{3\pi}{2} < x < \frac{2\pi}{2}$ , find:   $\frac{3\pi}{4} < \frac{x}{2} < \pi \rightarrow Q_2$

a)  $\sin 2x = 2 \sin \theta \cos \theta = 2 \left(-\frac{3}{5}\right) \left(\frac{4}{5}\right) = \boxed{\frac{-24}{25}} \rightarrow$  a)  $\frac{-24}{25}$

pos b)  $\sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}} = \sqrt{\frac{1 - 4/5}{2}} = \sqrt{\frac{1/5}{2}} = \sqrt{\frac{1}{5} \cdot \frac{1}{2}} = \sqrt{\frac{1}{10}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} \rightarrow$  b)  $\frac{\sqrt{10}}{10}$

neg c)  $\cos \frac{x}{2} = -\sqrt{\frac{1 + \cos x}{2}} = -\sqrt{\frac{1 + 4/5}{2}} = -\sqrt{\frac{9/5}{2}} = -\sqrt{\frac{9}{5} \cdot \frac{1}{2}} = -\sqrt{\frac{9}{10}} = -\frac{3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10} \rightarrow$  c)  $\frac{-3\sqrt{10}}{10}$

d)  $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} = \frac{2 \left(-\frac{3}{4}\right)}{1 - \left(-\frac{3}{4}\right)^2} = \frac{-6/4}{1 - 9/16} = \frac{-6/4}{7/16} = -\frac{6}{4} \cdot \frac{16}{7} = -\frac{3}{1} \cdot \frac{4}{7} = \boxed{\frac{-24}{7}} \rightarrow$  d)  $\frac{-24}{7}$

Find the exact value using the double or half angle identities:

e)  $\cos\left(\frac{7\pi}{3}\right) \quad \frac{7\pi}{3} \cdot \frac{1}{2} = \frac{7\pi}{6} = \cos\left(2 \cdot \frac{7\pi}{6}\right)$  \* use Double angle e)  $\frac{1}{2}$

f)  $\tan(105^\circ) \quad 105^\circ (2) = 210^\circ$  \* use half-angle f)  $-2 - \sqrt{3}$

$$\begin{aligned} \cos\left(2 \cdot \frac{7\pi}{6}\right) &= \cos^2\left(\frac{7\pi}{6}\right) - \sin^2\left(\frac{7\pi}{6}\right) \\ &= \left(-\frac{\sqrt{3}}{2}\right)^2 - \left(-\frac{1}{2}\right)^2 \\ &= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \boxed{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} f) \tan 105^\circ &= \tan\left(\frac{210^\circ}{2}\right) \\ &= \frac{1 - \cos 210^\circ}{\sin 210^\circ} \\ &= \frac{1 - \frac{-\sqrt{3}}{2}}{-1/2} = \frac{\frac{2}{2} + \frac{\sqrt{3}}{2}}{-1/2} = \frac{\frac{2 + \sqrt{3}}{2}}{-1/2} \\ &= \frac{2 + \sqrt{3}}{2} \cdot \frac{-2}{1} = -(2 + \sqrt{3}) = \boxed{-2 - \sqrt{3}} \end{aligned}$$