Warmup 3: Half Angle

Use the half angle identity to find exact value.

1. $\cos\frac{7\pi}{8}$

* $\frac{7\pi}{8}$ is in Quadrant 2 so cos is negative.

* Double angle: $\frac{7\pi}{8}\cdot 2 = \frac{14\pi}{8} = \frac{7\pi}{4}$

* Use half angle identity

$$\cos\left(\frac{7\pi/4}{2}\right) = -\sqrt{\frac{1+\cos\frac{7\pi}{4}}{2}}$$

$$= -\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{2}}$$

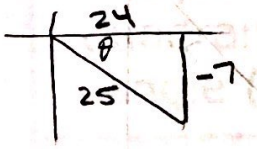
$$= -\sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2/1}} = -\sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}}$$

$$= -\sqrt{\frac{2+\sqrt{2}}{4}} = -\frac{\sqrt{2+\sqrt{2}}}{\sqrt{4}}$$

$$= \boxed{-\frac{\sqrt{2+\sqrt{2}}}{2}}$$

2. Given $\sin\theta = -\frac{7^{opp}}{25_{hyp}}$ and $270° < \theta < 360°$, find $\cos\frac{\theta}{2}$.

* $270° < \theta < 360°$ is in Quadrant 4



* $\frac{270°}{2} < \frac{\theta}{2} < \frac{360°}{2} \Rightarrow 135° < \frac{\theta}{2} < 180°$

Quadrant 2

* Use half-angle identity

$$-\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1+24/25}{2}} = -\sqrt{\frac{\frac{25}{25}+\frac{24}{25}}{2}}$$

$$= -\sqrt{\frac{49/25}{2/1}} = -\sqrt{\frac{49}{25}\cdot\frac{1}{2}} = -\sqrt{\frac{49}{50}}$$

$$= -\frac{\sqrt{49}}{\sqrt{50}} = -\frac{7}{5\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{-7\sqrt{2}}{10}}$$

$\sqrt{25}\cdot\sqrt{2}$