PreCalculus

Name Key

More Double and Half Angle Identities WS

$\frac{450^\circ}{2} = 225^\circ$

1. Use a double angle identity to find the exact value of $\cos 450^\circ$. $\cos 450^\circ = \cos(2 \cdot 225^\circ)$

$\cos^2\theta - \sin^2\theta = \cos^2(225^\circ) - \sin^2(225^\circ)$

$= \left(-\frac{\sqrt{2}}{2}\right)^2 - \left(-\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} - \frac{2}{4} = \boxed{0}$

2. Use a half angle identity to find the exact value of $\sin\frac{11\pi}{12}$. Q2: sin pos.

$\frac{11\pi}{12} \cdot \frac{2}{2} = \frac{11\pi}{6}$ $\sin\left(\frac{11\pi/6}{2}\right) = \sqrt{\frac{1-\cos 11\pi/6}{2}} = \sqrt{\frac{1-\sqrt{3}/2}{2}} = \sqrt{\frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{2} \cdot \frac{1}{2}}$

$= \sqrt{\frac{2-\sqrt{3}}{4}} = \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$

**** Show the expansion, substitution, and simplified answer as separate steps! ****

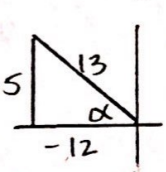
Use the given information to find the exact values of each trig function below:

* Show
① Expansion
② Substitution
③ Simplified Answer

α is in quadrant II and $\csc\alpha = \frac{13}{5}$ $\sin\alpha = \frac{5}{13}$

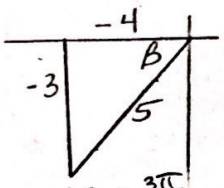
β is in quadrant III and $\cot\beta = \frac{4}{3}$ $\tan\beta = \frac{3}{4}$

θ is in quadrant IV and $\sec\theta = \frac{25}{7}$ $\cos\theta = \frac{7}{25}$



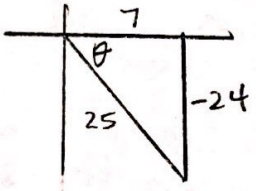
$\frac{\pi}{2} < \alpha < \pi$

$\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$



$\pi < \beta < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{\beta}{2} < \frac{3\pi}{4}$



$\frac{3\pi}{2} < \theta < 2\pi$

$\frac{3\pi}{4} < \frac{\theta}{2} < \pi$

3. $\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right) = \boxed{-\frac{120}{169}}$

4. $\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1-\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{16}{16}-\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{2} \cdot \frac{16^8}{7} = \boxed{\frac{24}{7}}$

5. $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{7}{25}\right)^2 - \left(-\frac{24}{25}\right)^2 = \frac{49}{625} - \frac{576}{625} = \boxed{\frac{-527}{625}}$

6. $\sin\frac{\beta}{2} = \sqrt{\frac{1-\cos\beta}{2}} = \sqrt{\frac{1-\left(\frac{-4}{5}\right)}{2}} = \sqrt{\frac{\frac{5}{5}+\frac{4}{5}}{2}} = \sqrt{\frac{\frac{9}{5}}{2}} = \sqrt{\frac{9}{5} \cdot \frac{1}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \boxed{\frac{3\sqrt{10}}{10}}$

Q2 Sin Pos

7. $\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+\frac{-12}{13}}{2}} = \sqrt{\frac{\frac{13}{13}-\frac{12}{13}}{2}} = \sqrt{\frac{1/13}{2}} = \sqrt{\frac{1}{13} \cdot \frac{1}{2}} = \sqrt{\frac{1}{26}} = \frac{1}{\sqrt{26}} \cdot \frac{\sqrt{26}}{\sqrt{26}} = \boxed{\frac{\sqrt{26}}{26}}$

Q1 cos pos

8. $\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{1-\frac{7}{25}}{-\frac{24}{25}} = \frac{\frac{25}{25}-\frac{7}{25}}{-\frac{24}{25}} = \frac{18/25}{-24/25} = \frac{18^3}{25} \cdot \frac{-25}{24_4} = \boxed{\frac{-3}{4}}$

Q tan neg

Answers: 1) 0 2) $\frac{\sqrt{2-\sqrt{3}}}{2}$ 3) $-\frac{120}{169}$ 4) $\frac{24}{7}$ 5) $-\frac{527}{625}$ 6) $\frac{3\sqrt{10}}{10}$ 7) $\frac{\sqrt{26}}{26}$ 8) $-\frac{3}{4}$