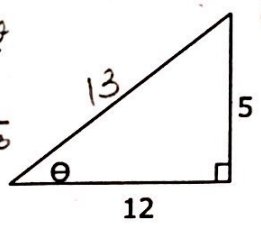
PreCalculus
Half Angle Trig Identities WS

Name Key

Use the figure to find the exact value of each trig function.



Q1 - All positive

1. $\cos\frac{\theta}{2}$ (Pos) $= \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+12/13}{2}} = \sqrt{\frac{25/13}{2}} = \sqrt{\frac{25}{13}\cdot\frac{1}{2}} = \sqrt{\frac{25}{26}} = \frac{5}{\sqrt{26}}\cdot\frac{\sqrt{26}}{\sqrt{26}} = \boxed{\frac{5\sqrt{26}}{26}}$

2. $\sin\frac{\theta}{2}$ (Pos) $= \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-12/13}{2}} = \sqrt{\frac{1/13}{2}} = \sqrt{\frac{1}{13}\cdot\frac{1}{2}} = \sqrt{\frac{1}{26}} = \frac{1}{\sqrt{26}}\frac{\sqrt{26}}{\sqrt{26}} = \boxed{\frac{\sqrt{26}}{26}}$

3. $\tan\frac{\theta}{2}$ (Pos) $= \frac{1-\cos\theta}{\sin\theta} = \frac{1-12/13}{5/13} = \frac{1/13}{5/13} = \frac{1}{13}\cdot\frac{13}{5} = \boxed{\frac{1}{5}}$

* use reciprocal functions

4. $\sec\frac{\theta}{2} = \boxed{\frac{\sqrt{26}}{5}}$

5. $\csc\frac{\theta}{2} = \boxed{\sqrt{26}}$

6. $\cot\frac{\theta}{2} = \boxed{5}$

7. $2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ $= 2\left(\frac{\sqrt{26}}{26}\right)\left(\frac{5\sqrt{26}}{26}\right) = \frac{10(26)}{676} = \frac{260}{676} = \boxed{\frac{5}{13}}$

8. $2\cos\frac{\theta}{2}\tan\frac{\theta}{2}$ $= 2\left(\frac{5\sqrt{26}}{26}\right)\left(\frac{1}{5}\right) = \frac{10\sqrt{26}}{130} = \boxed{\frac{\sqrt{26}}{13}}$

Use the half-angle identities to determine the exact values of each function. * on next page

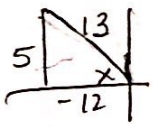
$\frac{30}{60} = \frac{1}{2}$ $\sin 112.5°$ $112.5(2) = 225°$

9. $\sin 112°30'$

10. $\cos\frac{\pi}{12}$

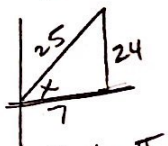
11. $\tan\frac{3\pi}{8}$

Find the exact values of each trig function using the half-angle identities.



12. Given ... $\sin x = \frac{50}{13}$, $\frac{\pi}{2} < x < \pi$ (Q2) ... find $\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1-(-12/13)}{2}} = \sqrt{\frac{13/13+12/13}{2}} = \sqrt{\frac{25/13}{2}} = \sqrt{\frac{25}{13}\cdot\frac{1}{2}} = \sqrt{\frac{25}{26}} = \frac{5}{\sqrt{26}}\cdot\frac{\sqrt{26}}{\sqrt{26}} = \boxed{\frac{5\sqrt{26}}{26}}$

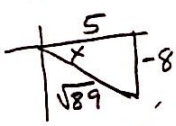
$\frac{\pi}{2}\cdot\frac{1}{2} = \frac{\pi}{4}$ $\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$ Q1



13. Given ... $\cos x = \frac{7a}{25}$, $0 < x < \frac{\pi}{2}$ (Q1) ... find $\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1+7/25}{2}} = \sqrt{\frac{25/25+7/25}{2}} = \sqrt{\frac{32/25}{2}} = \sqrt{\frac{32}{25}\cdot\frac{1}{2}} = \sqrt{\frac{16}{25}} = \boxed{\frac{4}{5}}$

$0 < x < \frac{\pi}{4}$ Q1

$\frac{\pi}{2}\cdot\frac{1}{2} = \frac{\pi}{4}$

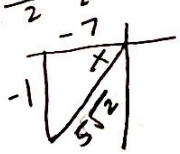


14. Given ... $\tan x = -\frac{8}{5}$, $\frac{3\pi}{2} < x < 2\pi$ (Q4) ... find $\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{1-5/\sqrt{89}}{-8/\sqrt{89}} = \frac{\sqrt{89}/\sqrt{89}-5/\sqrt{89}}{-8/\sqrt{89}} = \frac{\sqrt{89}-5}{\sqrt{89}}\cdot\frac{\sqrt{89}}{8} = -\frac{(\sqrt{89}-5)}{8} = \boxed{\frac{-\sqrt{89}+5}{8}}$

$5^2+(-8)^2=c^2$
$25+64=c^2$
$89=c^2$
$\sqrt{89}=c$

$\frac{3\pi}{4} < \frac{x}{2} < \pi$ Q2

$\frac{3\pi}{2}\cdot\frac{1}{2} = \frac{3\pi}{4}$



15. Given ... $\cot x = \frac{7a}{10}$, $\pi < x < \frac{3\pi}{2}$ (Q3 → cos = neg) ... find $\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{1+(-7/5\sqrt{2})}{2}} = -\sqrt{\frac{1+(-7\sqrt{2}/10)}{2}} = -\sqrt{\frac{10/10-7\sqrt{2}/10}{2}} = -\sqrt{\frac{10-7\sqrt{2}}{10}\cdot\frac{1}{2}} = -\sqrt{\frac{10-7\sqrt{2}}{20}} = -\frac{\sqrt{10-7\sqrt{2}}}{2\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = -\frac{\sqrt{5\cdot(10-7\sqrt{2})}}{10} = \boxed{-\frac{\sqrt{50-35\sqrt{2}}}{10}}$

$1^2+7^2=c^2$
$1+49=c^2$
$50=c^2$
$\sqrt{50}=c$
$5\sqrt{2}=c$

$\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$ Q2

$\frac{3\pi}{2}\cdot\frac{1}{2} = \frac{3\pi}{4}$

Half-Angle Trig Identity WS

9. $\sin(112°30') = \sin(112.5°)$ Q2: sin is Pos; $112.5(2) = 225°$

$\sin\left(\frac{225°}{2}\right) = \sqrt{\frac{1-\cos 225°}{2}}$

$= \sqrt{\frac{1-\left(\frac{-\sqrt{2}}{2}\right)}{2}} = \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}}$

$= \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{\sqrt{4}} = \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$

10. $\cos\frac{\pi}{12}$ Q1: cos is pos., $\frac{\pi}{12}\cdot 2 = \frac{\pi}{6}$

$\cos\left(\frac{\pi/6}{2}\right) = \sqrt{\frac{1+\cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{2}\cdot\frac{1}{2}}$

$= \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}} = \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$

11. $\tan\frac{3\pi}{8}$ Q1: tan is pos, $\frac{3\pi}{8}\cdot 2 = \frac{6\pi}{8} = \frac{3\pi}{4}$

$\tan\left(\frac{3\pi/4}{2}\right) = \frac{1-\cos\frac{3\pi}{4}}{\sin\frac{3\pi}{4}} = \frac{1-\frac{-\sqrt{2}}{2}}{\sqrt{2}/2} = \frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{\sqrt{2}/2}$

$= \frac{\frac{2+\sqrt{2}}{2}}{\sqrt{2}/2} = \frac{2+\sqrt{2}}{2}\cdot\frac{2}{\sqrt{2}} = \frac{(2+\sqrt{2})}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}+2}{2} = \boxed{\sqrt{2}+1}$

12. [illegible]