Half-Angle Identities

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

Your answer will NOT be both $\pm$. The signs of sin(u/2) and cos(u/2) depend on the quadrant in which angle (u/2) lies.

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u} = \pm\sqrt{\frac{1-\cos u}{1+\cos u}}$$

$105(2) = 210^\circ$

Ex. 1: Use the half-angle formula to find the exact values of …

Pos a) $\sin 105^\circ = \sin\left(\frac{210^\circ}{2}\right) = \sqrt{\frac{1-\cos 210^\circ}{2}} = \sqrt{\frac{1 - \frac{-\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2}}$

$= \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{\frac{2}{1}}} = \sqrt{\frac{2+\sqrt{3}}{2} \cdot \frac{1}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}} = \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$

Neg b) $\cos 105^\circ = \cos\left(\frac{210^\circ}{2}\right) = -\sqrt{\frac{1+\cos 210^\circ}{2}} = -\sqrt{\frac{1 + -\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{2}}$

$= -\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{\frac{2}{1}}} = -\sqrt{\frac{2-\sqrt{3}}{2} \cdot \frac{1}{2}} = -\sqrt{\frac{2-\sqrt{3}}{4}} = -\frac{\sqrt{2-\sqrt{3}}}{\sqrt{4}} = \boxed{-\frac{\sqrt{2-\sqrt{3}}}{2}}$

Neg c) $\tan 105^\circ = \tan\left(\frac{210^\circ}{2}\right) = \frac{1-\cos 210^\circ}{\sin 210^\circ} = \frac{1 - \frac{-\sqrt{3}}{2}}{-1/2}$

$= \frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{-1/2} = \frac{\frac{2+\sqrt{3}}{2}}{-1/2} = \frac{2+\sqrt{3}}{\cancel{2}} \cdot -\frac{\overset{(-1)}{\cancel{2}}}{1} = -1(2+\sqrt{3}) = \boxed{-2-\sqrt{3}}$

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$\frac{\pi}{8} \cdot 2 = \frac{2\pi}{8} = \frac{\pi}{4}$ $\qquad$ $\frac{\frac{\pi}{4}}{2/1} = \frac{\pi}{4} \cdot \frac{1}{2} = \frac{\pi}{8}$

Ex. 2: Use the half-angle formula to find the exact values of …

Pos a) $\sin \frac{\pi}{8} = \sin\left(\frac{\pi/4}{2}\right) = \sqrt{\frac{1-\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2}{2}-\frac{\sqrt{2}}{2}}{2}}$

$= \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2/1}} = \sqrt{\frac{2-\sqrt{2}}{2} \cdot \frac{1}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$

Pos b) $\cos \frac{\pi}{8} = \cos\left(\frac{\pi/4}{2}\right) = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}}$

$= \sqrt{\frac{2+\sqrt{2}}{2} \cdot \frac{1}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$

Pos c) $\tan \frac{\pi}{8}$

$= \frac{1-\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}} = \frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{\frac{2}{2}-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{\frac{2-\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2-\sqrt{2}}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{2}}$

$= \frac{(2-\sqrt{2})}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}-2}{2} = \boxed{\sqrt{2}-1}$

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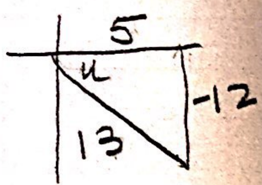
$$\frac{\frac{3\pi}{2}}{\frac{2}{1}} = \frac{3\pi}{2}\cdot\frac{1}{2} = \frac{3\pi}{4}$$

Pos

Ex. 3: Find the exact values of sin(u/2), cos(u/2) and tan(u/2) using the half-angle formulas.

neg neg

$$\cos u = \frac{5 \text{ adj}}{13 \text{ hyp}}, \quad \frac{3\pi}{2} < u < \frac{2\pi}{2}$$



$$\frac{3\pi}{4} < \frac{u}{2} < \pi$$

Q2

① $\sin\frac{u}{2} = \sqrt{\frac{1-\cos u}{2}}$

$$= \sqrt{\frac{1-5/13}{2}} = \sqrt{\frac{\frac{13}{13}-\frac{5}{13}}{2}} = \sqrt{\frac{\frac{8}{13}}{\frac{2}{1}}} = \sqrt{\frac{\cancel{8}^{4}}{13}\cdot\frac{1}{\cancel{2}}} = \sqrt{\frac{4}{13}} = \frac{2}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}} = \boxed{\frac{2\sqrt{13}}{13}}$$

② $\cos\frac{u}{2} = -\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1+\frac{5}{13}}{2}} = -\sqrt{\frac{\frac{13}{13}+\frac{5}{13}}{2}} = -\sqrt{\frac{\frac{18}{13}}{2}}$

$$= -\sqrt{\frac{\cancel{18}^{9}}{13}\cdot\frac{1}{\cancel{2}}} = -\sqrt{\frac{9}{13}} = -\frac{3}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}} = \boxed{\frac{-3\sqrt{13}}{13}}$$

③ $\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{1-\frac{5}{13}}{-\frac{12}{13}} = \frac{\frac{13}{13}-\frac{5}{13}}{-\frac{12}{13}} = \frac{\frac{8}{13}}{-12/13}$

$$= \frac{\cancel{8}^{2}}{\cancel{13}}\cdot -\frac{\cancel{13}}{\cancel{12}_{3}} = \boxed{-\frac{2}{3}}$$

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