

STANDARD FORM: $(x-h)^2 + (y-k)^2 = r^2$

GENERAL FORM: $ax^2 + by^2 + cx + dy + e = 0$

1. Convert to general form: $x^2 + (y+4)^2 = 9$.

$$\begin{aligned}x^2 + (y+4)(y+4) &= 9 \\x^2 + y^2 + 4y + 4y + 16 &= 9 \\x^2 + y^2 + 8y + 16 - 9 &= 9 - 9 \\x^2 + y^2 + 8y + 7 &= 0\end{aligned}$$

2. Convert to general form: $(x+1)^2 + (y-4)^2 = 8$.

$$\begin{aligned}(x+1)(x+1) + (y-4)(y-4) &= 8 \\x^2 + x + x + 1 + y^2 - 4y - 4y + 16 &= 8 \\x^2 + 2x + y^2 - 8y + 17 &= 8 \\x^2 + 2x + y^2 - 8y + 9 &= 0 \\x^2 + y^2 + 2x - 8y + 9 &= 0\end{aligned}$$

CONVERTING FROM GENERAL TO STANDARD FORM!!!

If the quadratic equation isn't in the standard form for a circle, we must first **complete the square** to get it in the correct form.

$$\left(\frac{16}{2}\right)^2 = 64 \quad \left(\frac{22}{2}\right)^2 = 121$$

Steps to complete the square.

First, prepare the terms:

- Group x^2 and leave a space.
- Group y^2 and leave a space.
- Move the constant and leave 2 spaces.

Then, complete the square:

- $\frac{1}{2}$ the linear term and square it.
- Add to both sides.
- Do this for both x and y.
- Factor and simplify.

1. $x^2 + y^2 + 16x - 22y - 20 = 0$

$$x^2 + 16x + 64 + y^2 - 22y + 121 = 20 + 64 + 121$$

$$(x+8)^2 + (y-11)^2 = 205$$

2. $x^2 + y^2 - 12x + 8y + 32 = 0$

$$x^2 - 12x + 36 + y^2 + 8y + 16 = -32 + 36 + 16$$

$$(x-6)^2 + (y+4)^2 = 20$$

3. $x^2 + y^2 + 2x - 15 = 0$

$$x^2 + 2x + 1 + y^2 = 15 + 1$$

$$(x+1)^2 + y^2 = 16$$