

Warmup 4: Use the sum or difference formula

to find the exact value of $\tan \frac{19\pi}{12} = \frac{3\pi}{12} + \frac{16\pi}{12}$

$$\tan \frac{19\pi}{12} = \tan \frac{\pi}{4} + \frac{4\pi}{3}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\frac{\tan \frac{\pi}{4} + \tan \frac{4\pi}{3}}{1 - \tan \frac{\pi}{4} \cdot \tan \frac{4\pi}{3}} = \frac{1 + \sqrt{3}}{1 - 1 \cdot \sqrt{3}} = \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \cdot \frac{(1 + \sqrt{3})}{(1 + \sqrt{3})}$$

$$= \frac{1 + \sqrt{3} + \sqrt{3} + 3}{1 - 3} = \frac{4 + 2\sqrt{3}}{-2} = \boxed{-2 - \sqrt{3}} \text{ or } \boxed{-\sqrt{3} - 2}$$