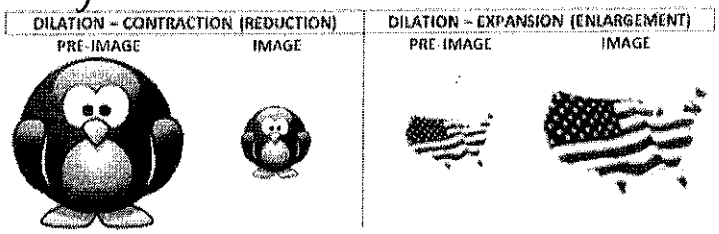
Geometry
Dilations

Date Key

Dilation: A transformation that enlarges or reduces the size of an object.



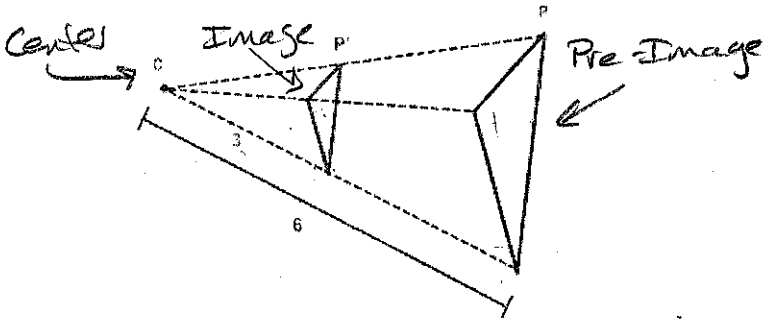
Scale Factor

-The preimage is enlarged or reduced by a Scale factor (k)

$K = \frac{\text{image}}{\text{preimage}}$

-The scale factor is determined by the distance from the center (C)

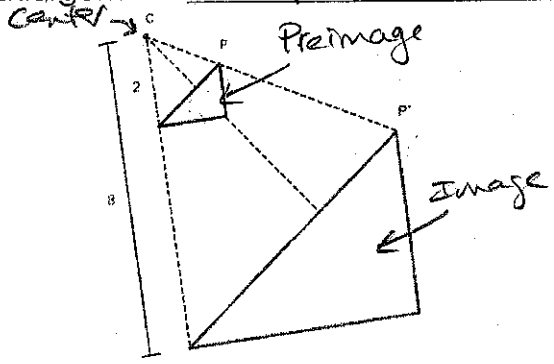
Reduction: $0 < K < 1$



$K = \frac{3}{6} = \frac{1}{2}$

Reduction or Enlargement

Enlargement: $K > 1$



$K = \frac{8}{2} = 4$

Reduction or Enlargement

Notation

$D_{C,K}(x,y) \rightarrow (Kx, Ky)$

C is the center

K is the value of the scale factor

Dilation Properties

Dilation is NOT an isometric transformation so its properties differ from the ones we saw with reflection, rotation and translation. The following properties are preserved between the pre-image and its image when dilating:

Angle Measure - Angles stay the same.

Parallelism - Things that were parallel are still parallel.

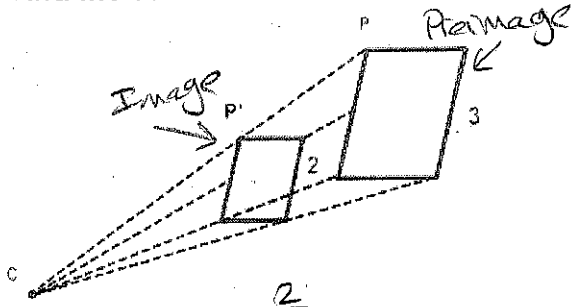
Collinearity - Points on a line remain on the line.

Distance IS NOT PRESERVED!!!

After a dilation, the pre-image and image have the same shape but not the same size.

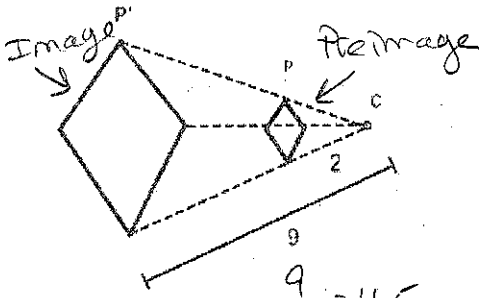
$K = \frac{\text{image}}{\text{Preimage}}$

Find the scale factor and determine if the dilation is an enlargement or a reduction.



$K = \frac{2}{3}$

Reduction or Enlargement



$K = \frac{9}{2} = 4.5$

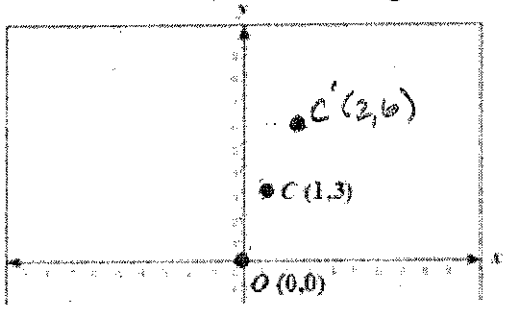
Reduction or Enlargement

Dilations of points and segments in the Coordinate Plane when the Origin is the Center

For a dilation to maintain its proportionality of sides, the two variables must be multiplied by a constant value, k, which is the scale factor.

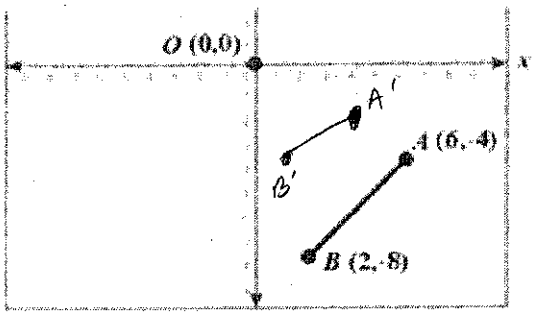
$D_{O,k}(x, y) = (kx, ky)$

Example 1: A dilation of 2 with center of dilation O, the origin.



Preimage C (1, 3) → $(2 \cdot 1, 2 \cdot 3)$ Image C'(2, 6)

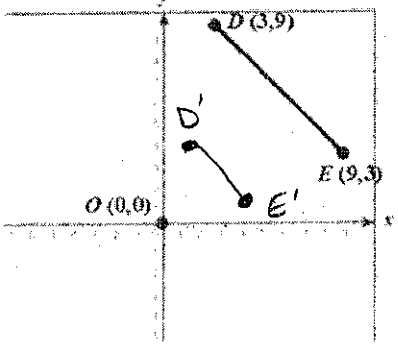
Example 2: A dilation of 1/2 with center of dilation O, the origin.



Preimage A (6, -4) $(\frac{1}{2} \cdot 6, \frac{1}{2} \cdot -4)$ Image A'(3, -2)

Preimage B (2, -8) $(\frac{1}{2} \cdot 2, \frac{1}{2} \cdot -8)$ Image B'(1, -4)

Example 3: A dilation of 1/3 with center of dilation O, the origin.

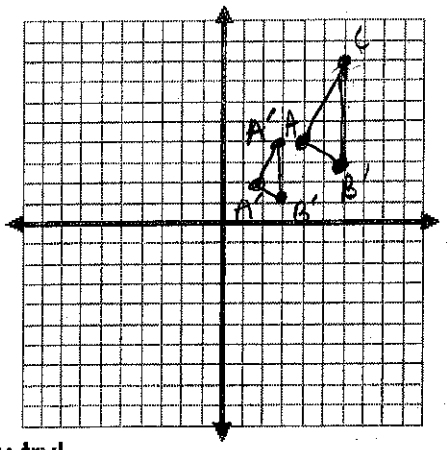


Preimage D (3, 9) $(\frac{1}{3} \cdot 3, \frac{1}{3} \cdot 9)$ Image D'(1, 3)

Preimage E (9, 3) $(\frac{1}{3} \cdot 9, \frac{1}{3} \cdot 3)$ Image E'(3, 1)

$K = \frac{\text{Image}}{\text{Preimage}}$

Dilations of polygons in the Coordinate Plane when the Origin is the Center

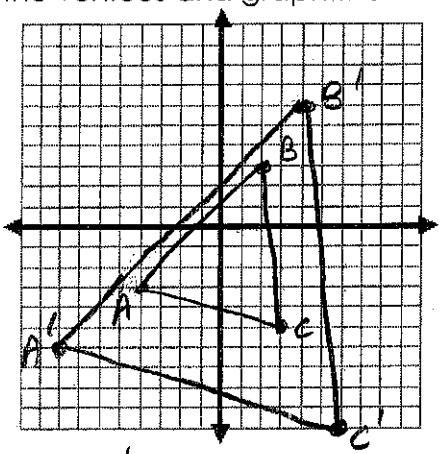


Dilation Notation: $D_{0,\frac{1}{2}} \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$

Preimage	Image	
A(4, 4)	A'(2, 2)	$\frac{2}{4} = \frac{1}{2}$
B(6, 3)	B'(3, 1.5)	
C(6, 8)	C'(3, 4)	$\frac{3}{6} = \frac{1}{2}$

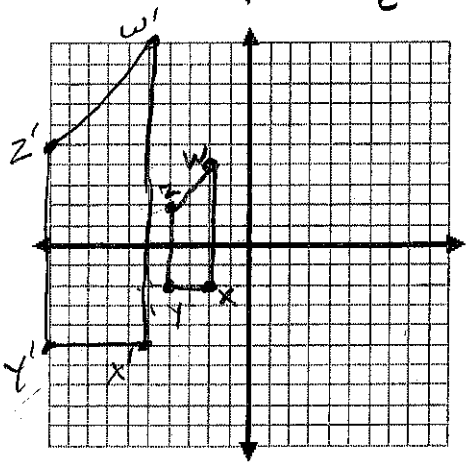
You try!

Find the vertices and graph the image after a dilation centered at the origin with the given scale factor



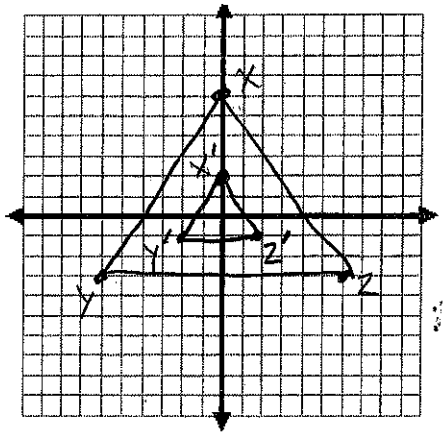
Dilation Notation: $D_{0,2}\ (x, y) \rightarrow (2x, 2y)$

Preimage	Image
A (-4, -3)	A'(-8, -6)
B (2, 3)	B' (4, 6)
C (3, -5)	C' (6, -10)



Dilation Notation: $D_{0,2.5}\ (x, y) \rightarrow (2.5x, 2.5y)$

Preimage	Image
W (-2, 4)	W'(-5, 10)
X (-2, -2)	X'(-5, -5)
Y (-4, -2)	Y' (-10, -5)
Z (-4, 2)	Z' (-10, 5)



Dilation Notation: $D_{0,\frac{1}{3}}\ (x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$

Preimage	Image
X (0, 6)	X' (0, 2)
Y (-6, -3)	Y' (-2, -1)
Z (6, -3)	Z' (2, -1)